

# Formulations of General Relativity

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4 Lectures one hour each Monday and Tuesday

The goal is to collect all known **Lagrangian formulations** of General Relativity, with emphasis on four spacetime dimensions

**Why bother?**

GR person



1) Physics is independent of any choice of variables one uses to describe it. We have the formulation that is most developed (metric). Why develop unnecessary alternatives?

String theorist



2) GR is the unique low energy Lorentz-invariant interacting theory of massless spin two particles. This statement is independent of any Lagrangian formulation of it. Lagrangian is irrelevant, everything follows from amplitudes

## Answers to GR person objection

Practical: some of the formulations exhibit much less non-linearity than the Einstein-Hilbert formulation. It may be (is) **easier to compute** in one of the alternatives

### Conceptual:

“*There is always another way to say the same thing that doesn't look at all like the way you said it before. I don't know what the reason for this is. I think it is somehow a representation of the simplicity of nature? Perhaps a thing is simple if you can describe it fully in several different ways without immediately knowing that you are describing the same thing.*”

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Richard Feynman, *Nobel Lecture*, 1965

One will never fully appreciate the beauty of GR without absorbing all of its equivalent but not obviously so reformulations

““ *Theories of the known, which are described by different physical ideas may be equivalent in all their predictions and are hence scientifically indistinguishable. However, they are not psychologically identical when trying to move from that base into the unknown. For different views suggest different kinds of modifications which might be made and hence are not equivalent in the hypotheses one generates from them in ones attempt to understand what is not yet understood. I, therefore, think that a good theoretical physicist today might find it useful to have a wide range of physical viewpoints and mathematical expressions of the same theory available to him.* ””

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Richard Feynman, *Nobel Lecture*, 1965

We all believe that **GR is just an effective theory**, and as such just an approximation to some more fundamental description

But it may well be that **it contains hints** as to where to go in the search for this **more fundamental theory**. Such a hint may only be apparent in one of its formulations

GR is a dynamical theory of geometry, but there are many different types of geometry apart from the metric one. If the sought fundamental theory is geometric (?), which of the types of geometry we can use to rephrase GR is holding the key?

## Answers to string theory person objection

It is true we don't need a Lagrangian to do gravity as a theory of interacting gravitons in Minkowski space - amplitudes

But at the moment there is no analogous story around any other background, not even the constant curvature one (dS).

The only thing we can do is to expand a Lagrangian

More fundamentally, one wants more than a collection of perturbative descriptions - one wants a non-perturbative formulation of the theory. Lagrangian is the simplest way

## The fundamental question that is my personal motivation

There is a metric apparently filling all of the Universe. GR describes dynamics of this metric **if it is assumed to be there.**

But it does not answer the question why it is there in the first place. My hope is that by thinking about geometry(ies) of GR one can get closer to answering **“Why non-zero metric?”**

Einstein, one year before death, speaking to a group of Wheeler's students:

“There is much reason to be attracted to a theory with no space and no time. But nobody has any idea how to build it.”

## These lectures will cover:

### I) Metric and related formulations:

Einstein-Hilbert

First-order Palatini

Pure connection Eddington-Schroedinger

### II) Tetrad and related formulations:

First-order Einstein-Cartan

MacDowell-Mansouri, Stelle-West

Pure spin connection

### III) BF and related:

BF plus constraints

Pure spin connection in closed form

BF plus potential

### IV) Chiral formulations of 4D GR