QUANTUM SYSTEM IDENTIFICATION WITH LIMITED RESOURCES

DANIEL BURGARTH & KAZUYA YUASA
(ABERYSTWYTH) (WASEDA)

NOTTINGHAM, 25/6/12

OUTLINE

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* MOTIVATION

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* IDENTIFIABILITY WITH LIMITED RESOURCES

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* MOTIVATION
* IDENTIFIABILITY WITH LIMITED RESOURCES
* TOPOLOGICAL KNOWLEDGE

OUTLINE

* MOTIVATION
* IDENTIFIABILITY WITH LIMITED RESOURCES
* TOPOLOGICAL KNOWLEDGE
  → INFECTION

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* TOPOLOGICAL KNOWLEDGE
  → INFECTION
* INDIRECT ESTIMATION & CONTROL

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* INDIRECT ESTIMATION & CONTROL
* OUTLOOK & CONCLUSIONS

MOTIVATION

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→ GENERIC FRAMEWORK : ULTIMATE LIMIT OF PROTOCOLS, GUIDANCE TO "MISSING PARTS"

SETUP

SETUP (ANALOGOUS TO SONTAG)

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THINGS WE CAN MEASURE	M_ℓ

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* WHICH SYSTEMS $\hat{\sigma}$ ARE INDISTINGUISHABLE ?

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("ANONYMOUS TRANSLATOR")

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$\boxed{\text{THEOREM: IF } \sigma \text{ AND } \hat{\sigma} \text{ CONTROLLABLE, INDISTINGUISHABLE} \Leftrightarrow \text{UNITARILY RELATED}}$

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$\rightarrow$ CONTROLLABLE: $L = \langle H_0, H_1, \ldots, H_k \rangle_{[\cdot,\cdot]} = su(d)$

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* FORMULATE INDISTINGUISHABILITY ALGEBRAICALLY

$$(M_\ell | \mathcal{L}_{k_1} \mathcal{L}_{k_2} \cdots \mathcal{L}_{k_j} | \varrho_i) = (\hat{M}_\ell | \hat{\mathcal{L}}_{k_1} \hat{\mathcal{L}}_{k_2} \cdots \hat{\mathcal{L}}_{k_j} | \hat{\varrho}_i)$$

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TAKING PRE-KNOWLEDGE INTO ACCOUNT

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⤳ EACH KNOWN X RESTRICTS THE POSSIBLE W'S

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→ ALMOST EVERYTHING IDENTIFIABLE WITH FEW MEASUREMENTS.

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GRAPH INFECTION

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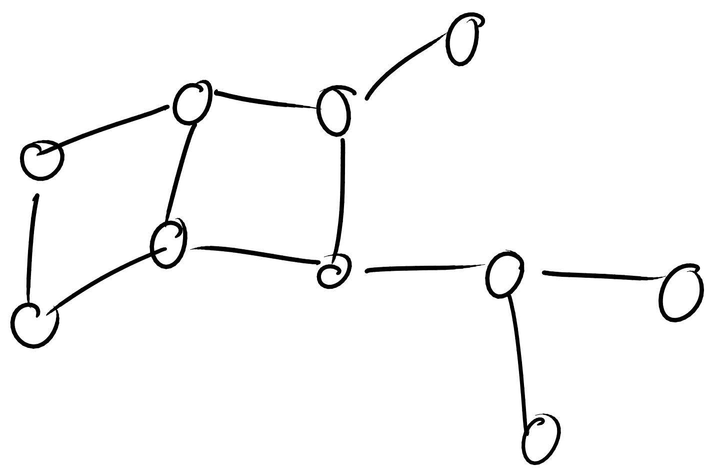

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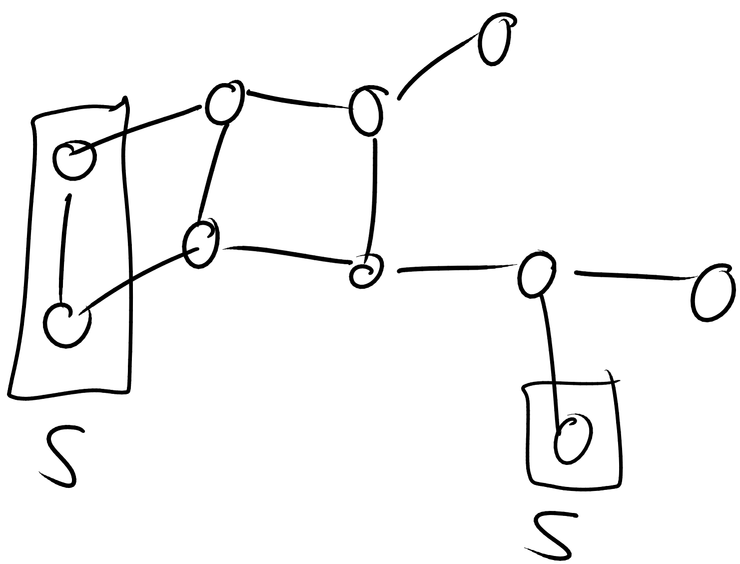

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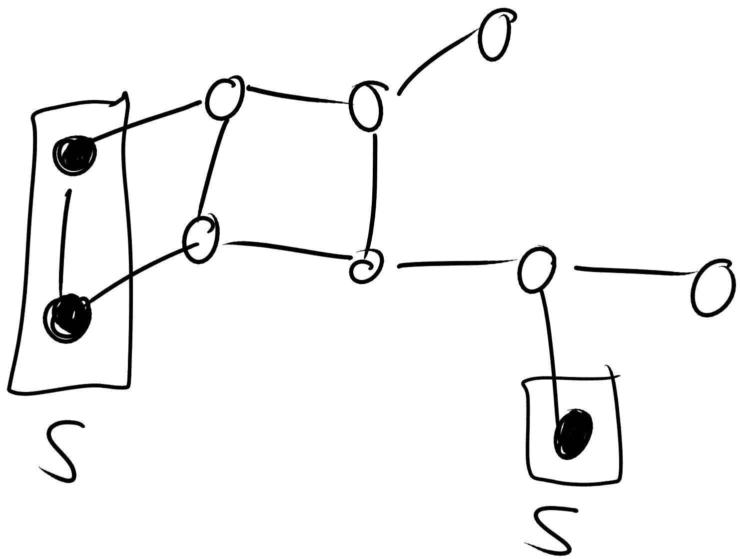

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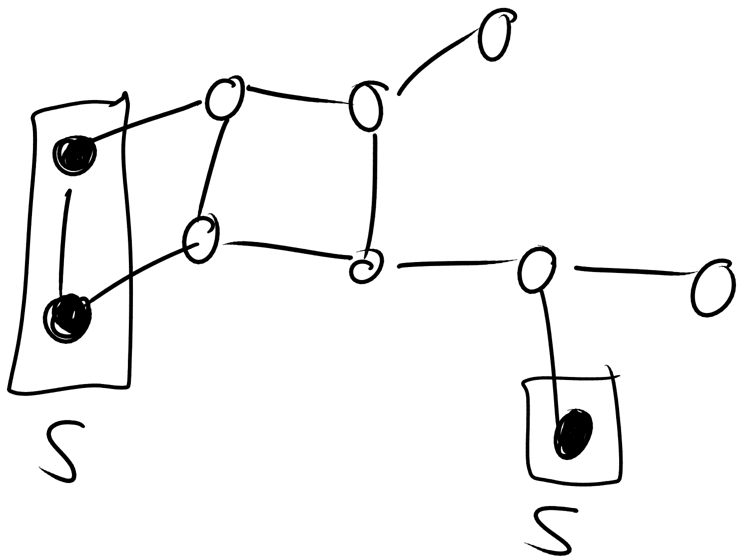

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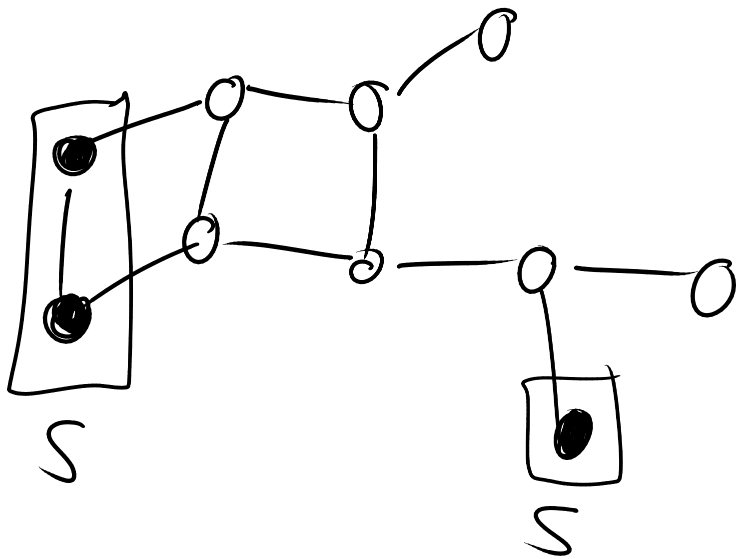

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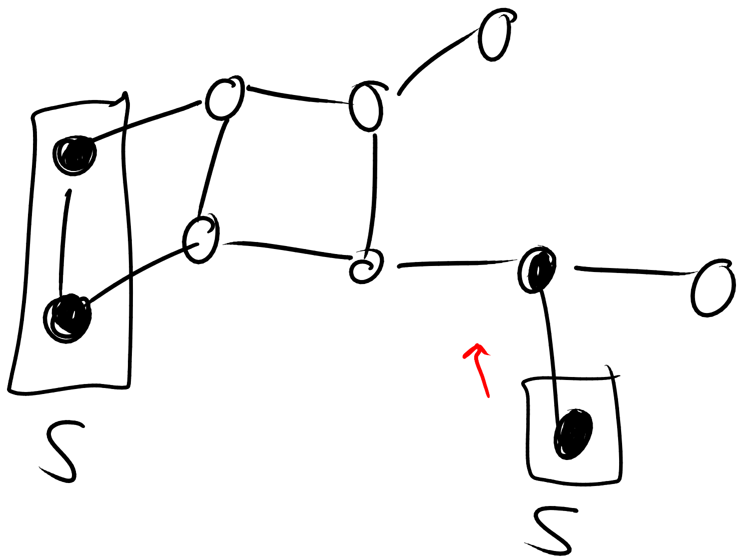

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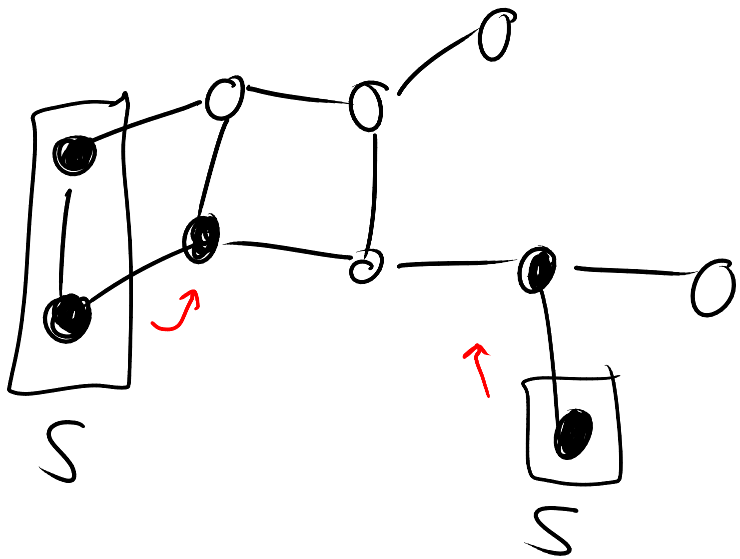

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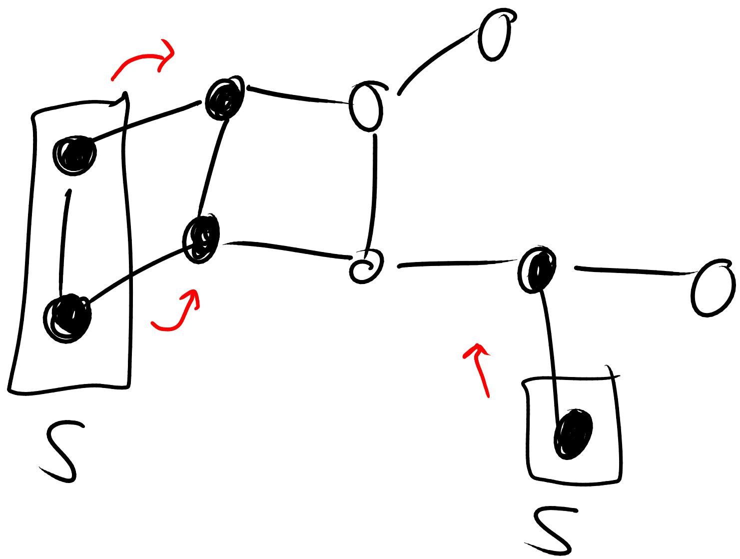

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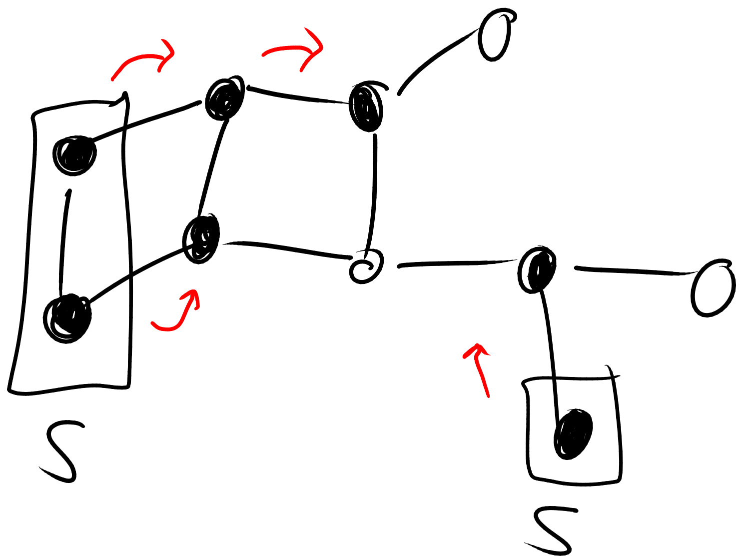

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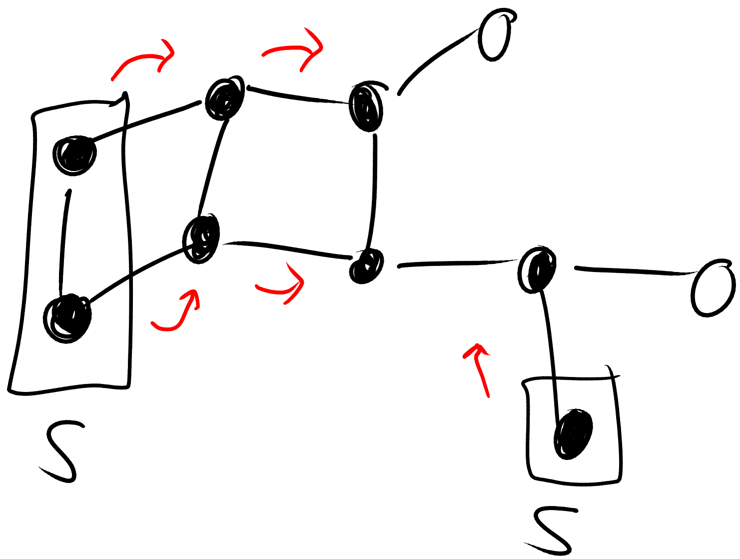

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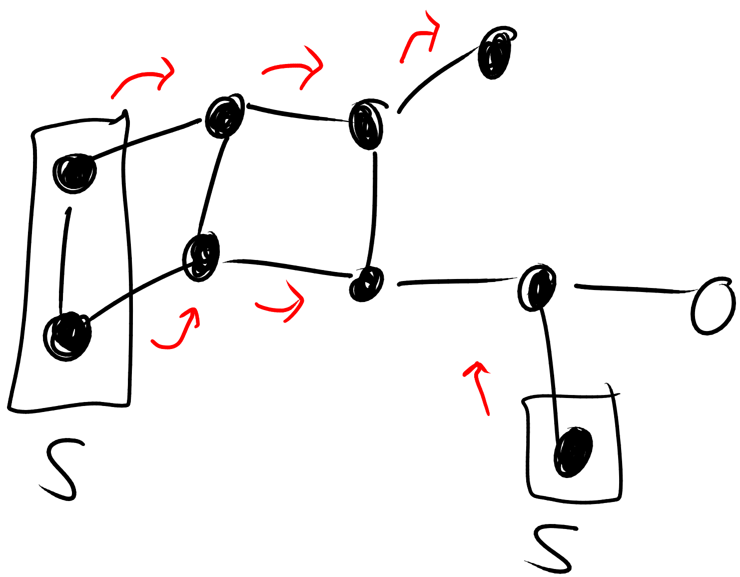

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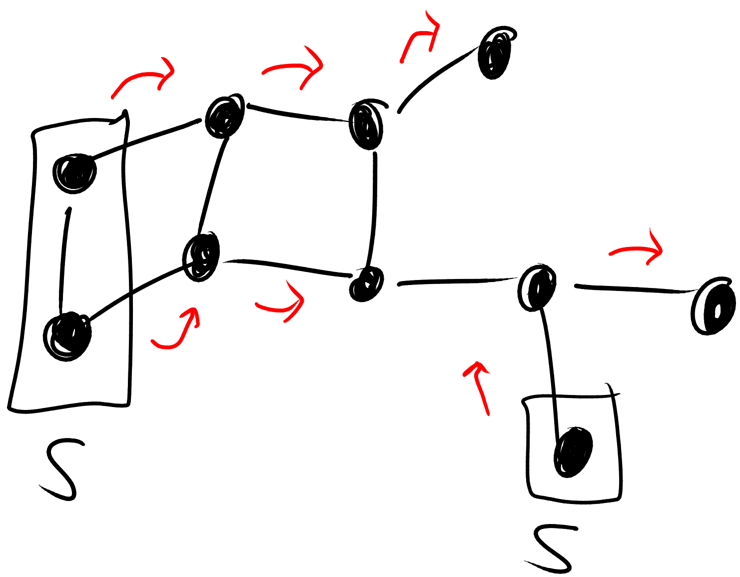

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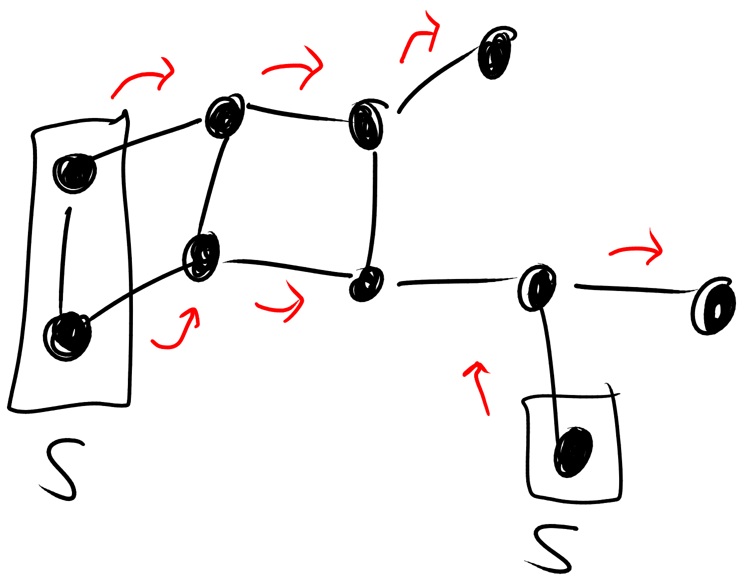

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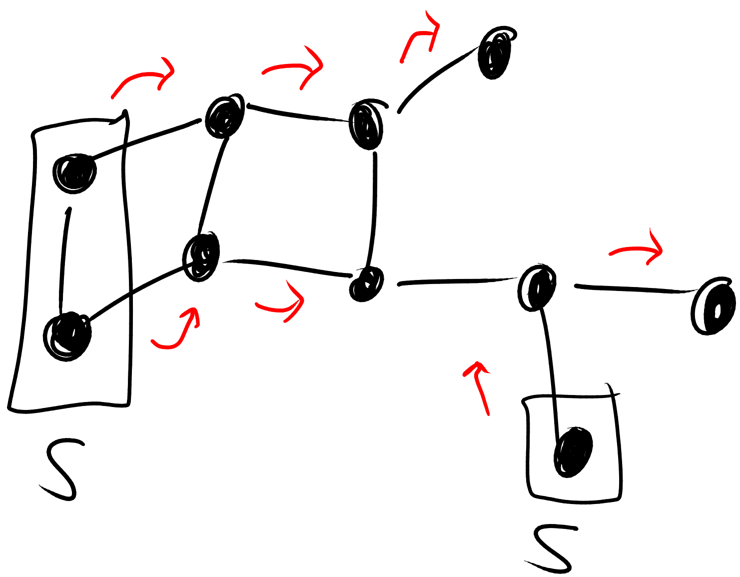

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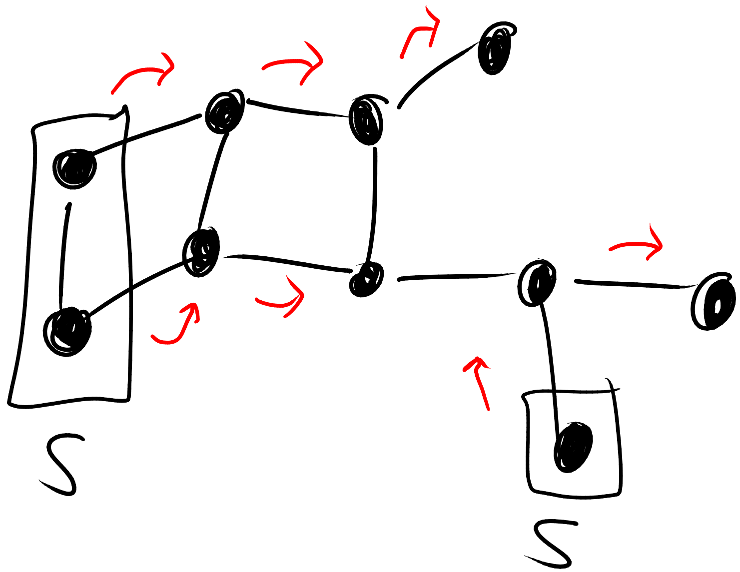

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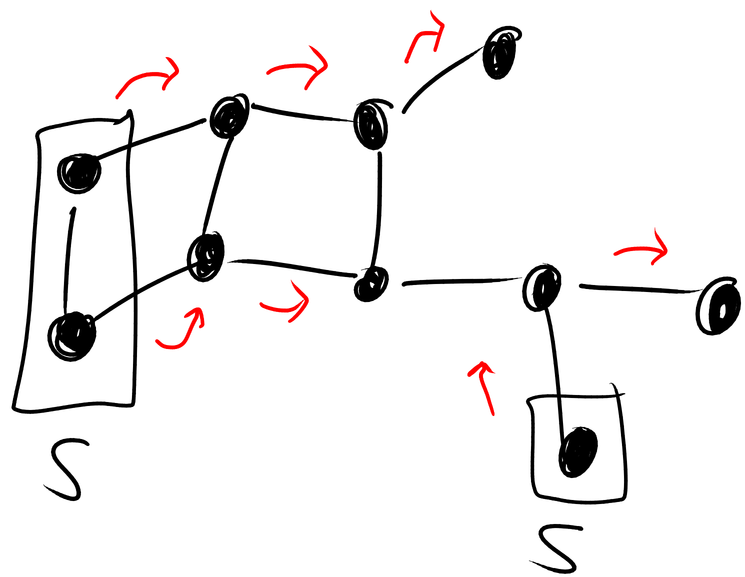

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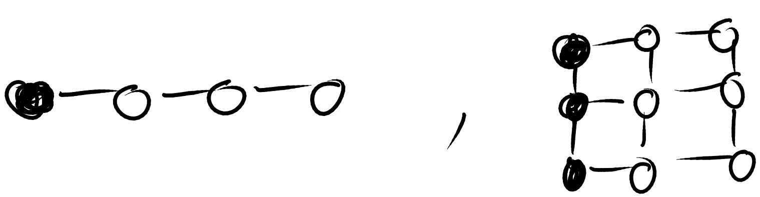

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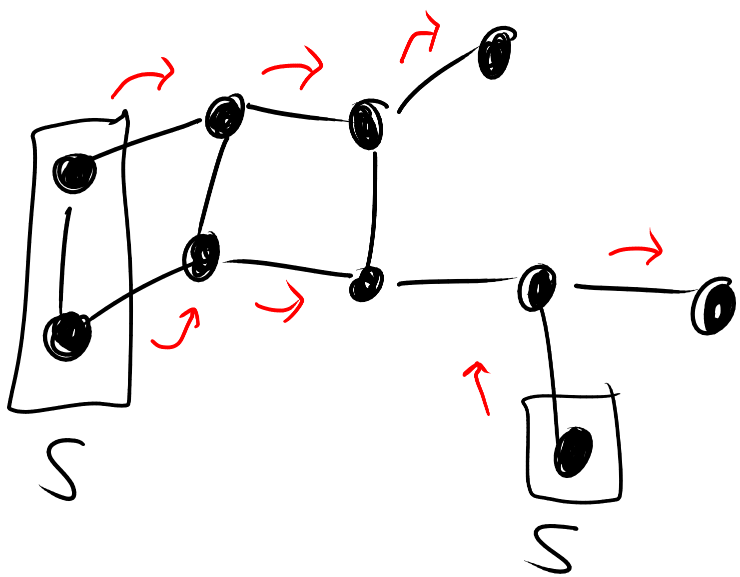

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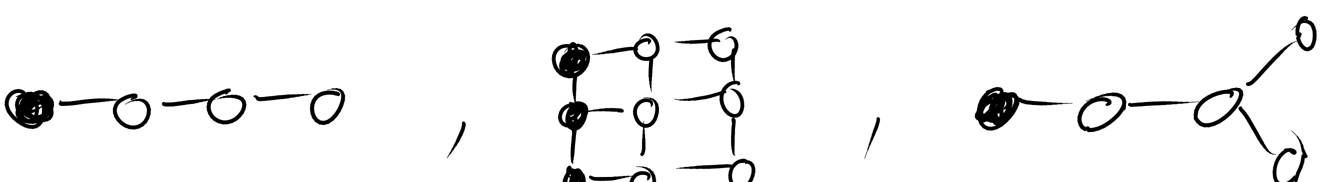

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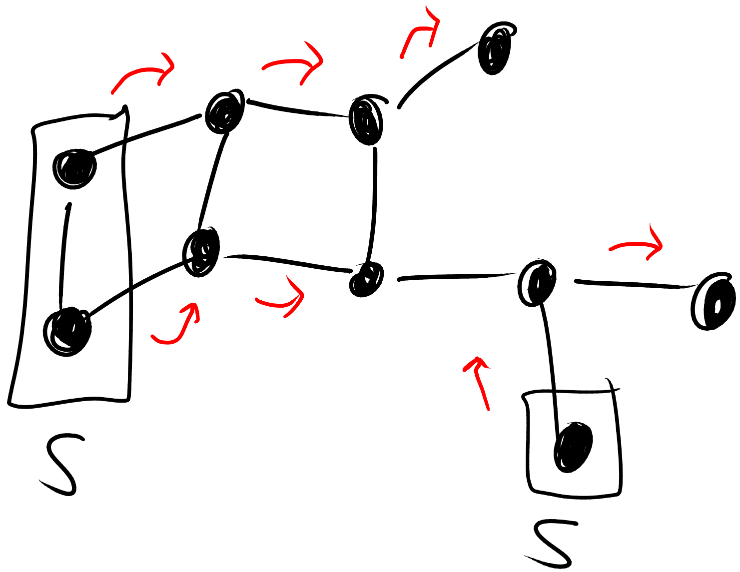

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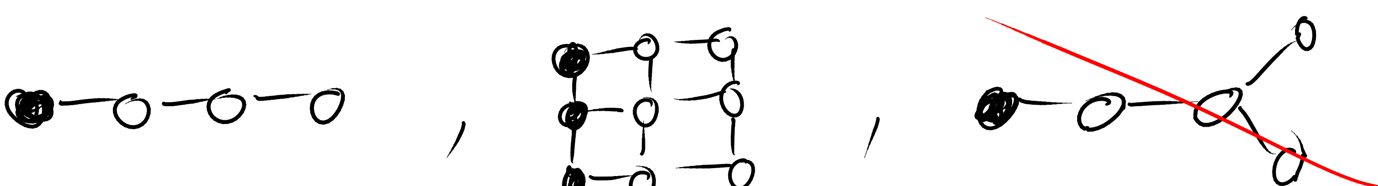

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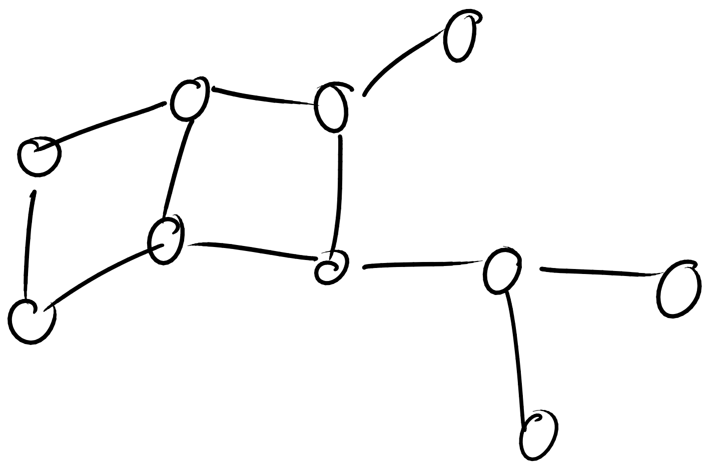

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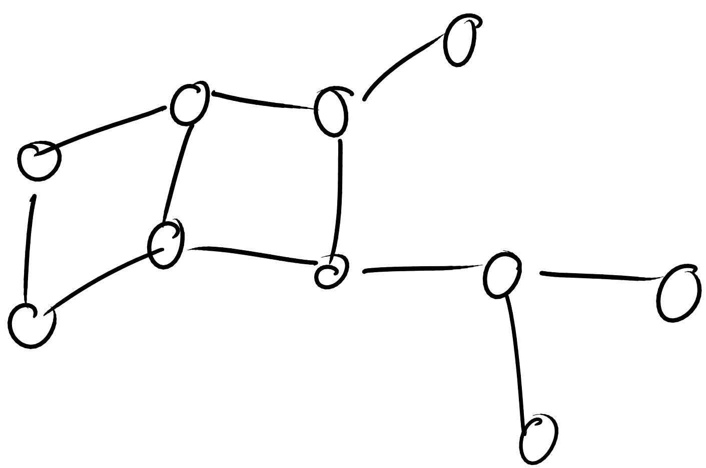

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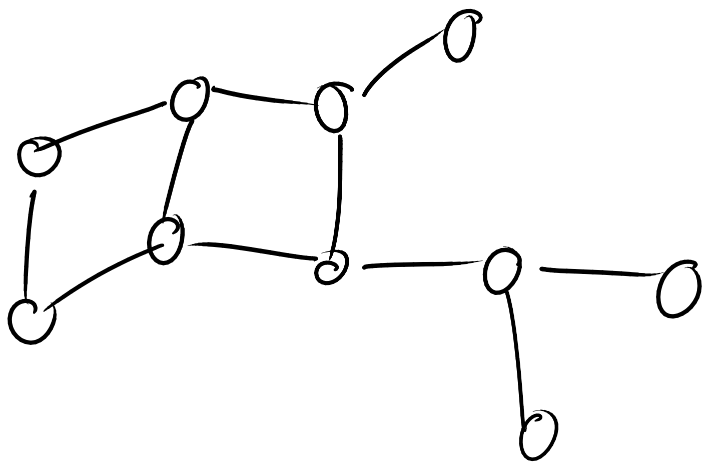

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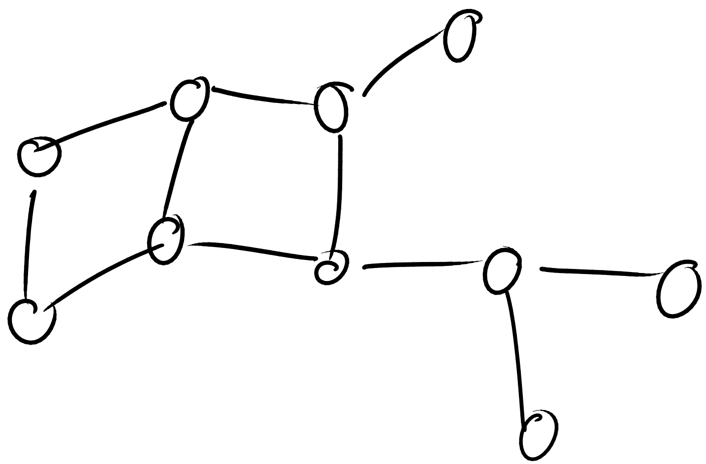

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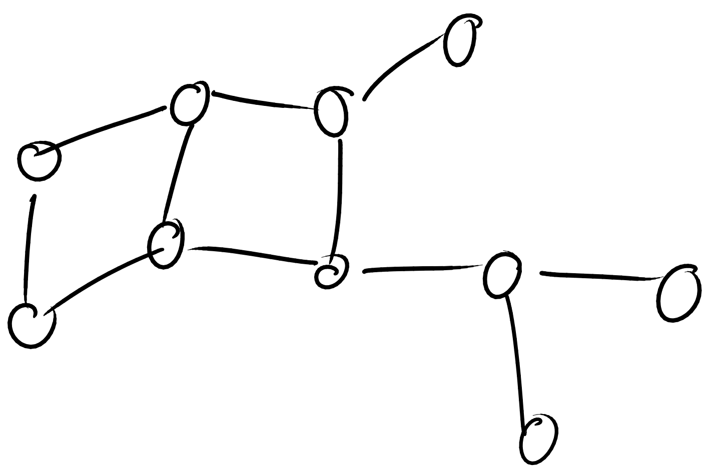

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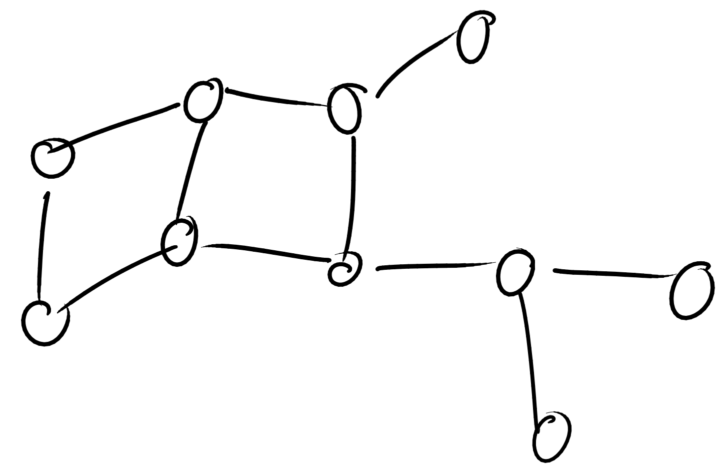

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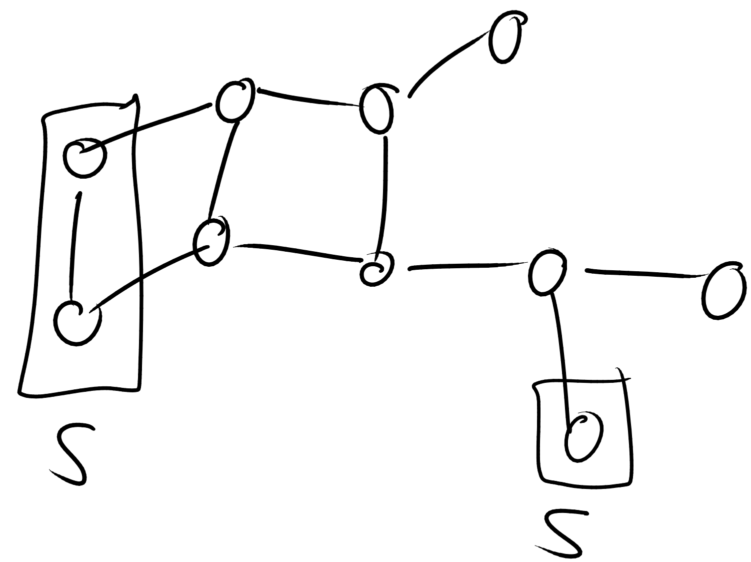

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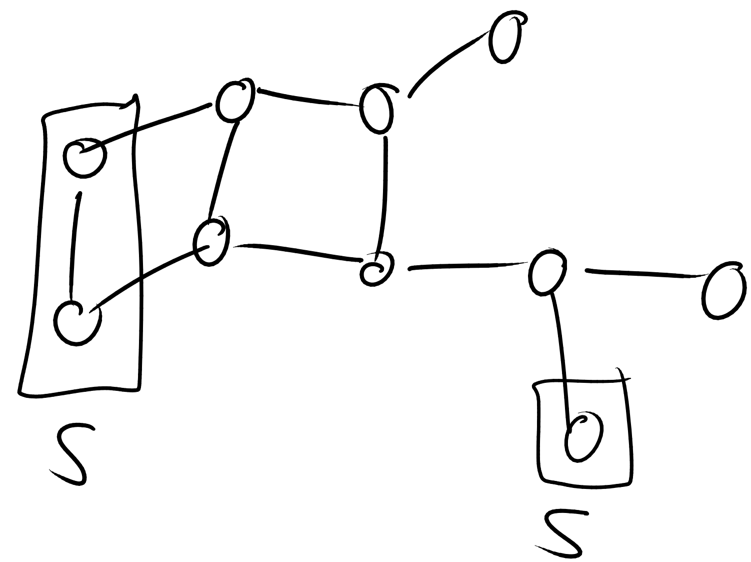

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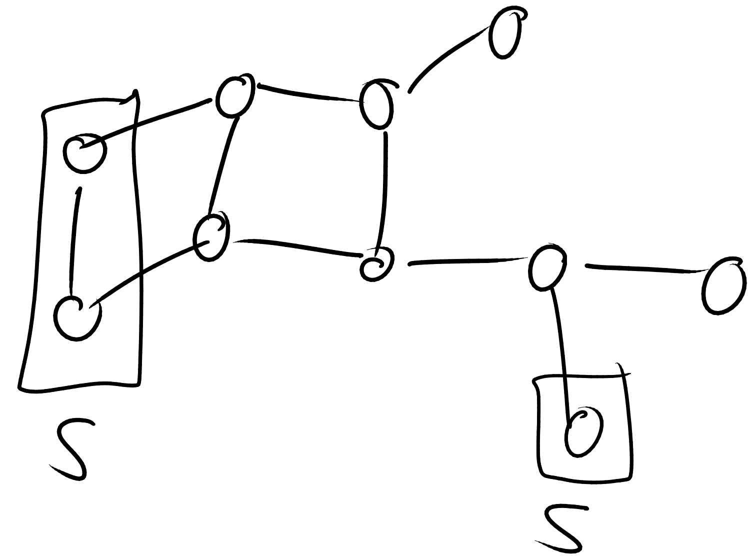

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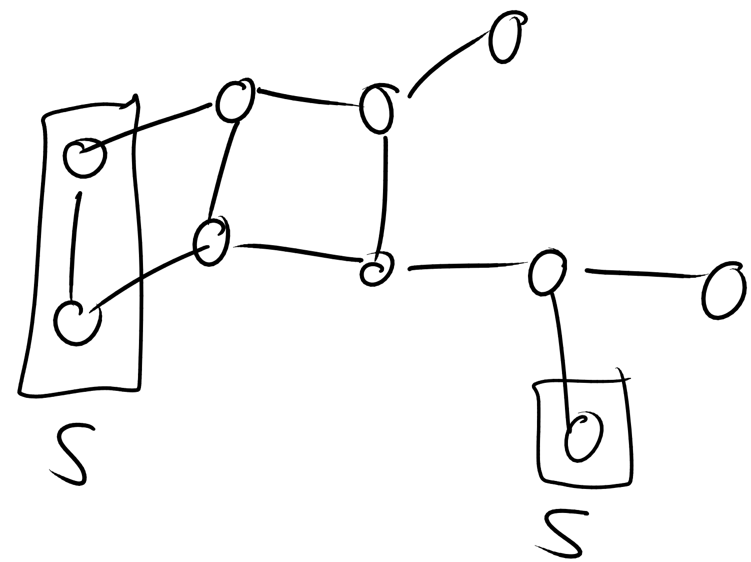

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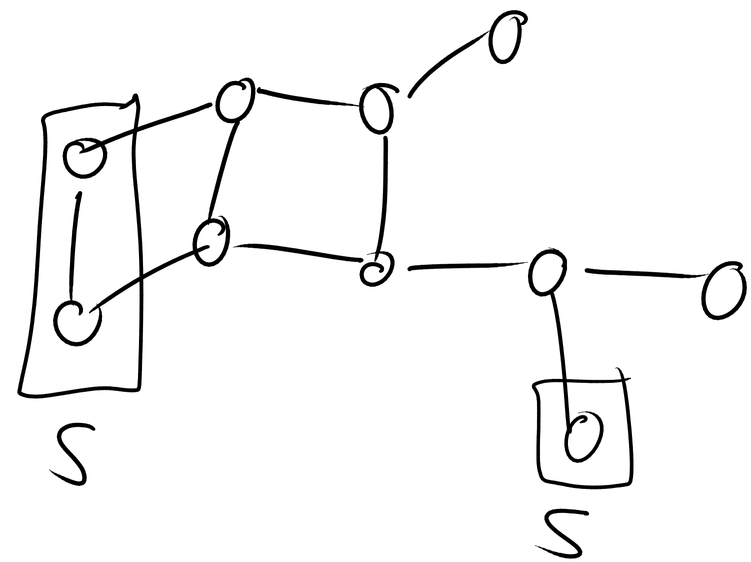

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CONCLUSIONS

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