Hyperbolic geometry for 3d gravity

3. More on hyperbolic surfaces

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Tangent and cotangent to Teichmüller

Lemma : given a vector field v on S, the corresponding variation of the complex structure on S vanishes iff v is holomorphic. This first-order variation is determined by the *Beltrami differential* of v, of the form $\bar\partial v \simeq b d\bar z(\partial/\partial z) \simeq b d\bar z/dz$.

Thus, Beltrami differential form the tangent space $T_c\mathcal{T}$ at a point $c \in \mathcal{T}$.

Given a Beltrami differential $b d\bar z/dz$ and a QHD $f dz^2$, their product $bf|dz|^2$ can be integrated over S. This pairing identifies the space of QHD with $T^*\mathcal{T}$.

There is a natural almost-complex structure on $\mathcal{T}$, defined through $T^*\mathcal{T}$ by $Jq = iq$. It is in fact complex : $\forall c \in \mathcal{T}, \exists U \ni x, \phi : U \rightarrow \mathbb{C}^N$ sending J to the multiplication by i.

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The Weil-Petersson metric

Given two QHD $q = fdz^2$ and $q' = f'dz^2$, one can consider the product $q\overline{q'} = f\overline{f'}|dz|^4$, and divide it by the hyperbolic metric $\rho|dz|^2$ obtained by solving the Liouville equation. The real part can be integrated over S.
This defines a scalar product on $T_c^*\mathcal{T}$, and a metric on $\mathcal{T}$, the Weil-Petersson metric g_{WP}.
Thm (Weil, '50) : g_{WP} is Kähler.
That is, g_{WP} is compatible with the complex structure J on QHD, and $g_{WP}(\cdot, J\cdot)$ is a symplectic form on $\mathcal{T}$.
Note : this theorem is not so trivial, in the sequel we will see one (more recent) approach to the proof. g_{WP} is the natural metric on the space of complex structures, however its definition needs the hyperbolic metric (and the solution of the Liouville equation).

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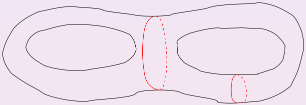
Multicurves

A *multicurve* is a disjoint union of simple closed curves.

Given a hyperbolic metric, each curve can be realized as a geodesic.

A weighted multicurve comes with positive weights on the curves.

Multicurves can be complicated.



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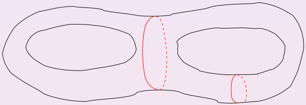

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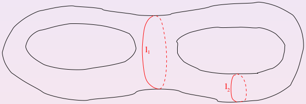

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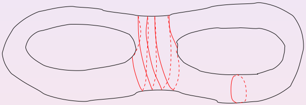

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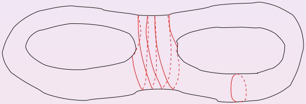

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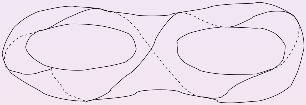
Measured laminations

Consider a (geodesic) weighted multicurve $(c_i, l_i)_{i=1,\cdots,n}$.

The c_i form a *lamination* and the l_i define a *transverse measure* : gives a total weight to γ, transverse to the c_i.

This gives a topology to the space of weighted multicurves.

Its completion is the space of *measured laminations* $\mathcal{ML}_S$.



$\mathcal{ML}_S$ is topologically a ball of dimension $6g - 6$.

Note : the notion of measured lamination is *topological*, although a given measured lamination can be realized uniquely as a geodesic measured lamination for any hyperbolic metric on S.

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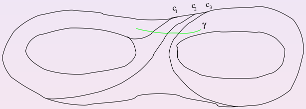

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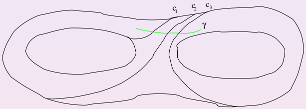

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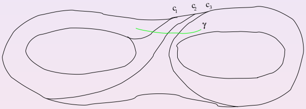

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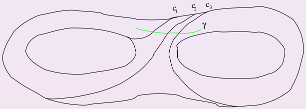

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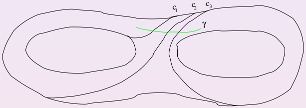

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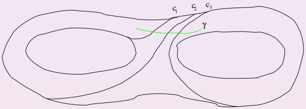

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Measured laminations as cotangent vectors

Let (c_i, l_i) be a weighted multicurve. It can be realised uniquely as a geodesic multicurve in any hyperbolic metric. Its length L defines a function on $\mathcal{T}$. $dL \in \mathcal{T}^*_c$, for all $c \in \mathcal{T}$.

Thm (Thurston) : this extends continuously to a one-to-one map from $\mathcal{ML}_S$ to $T^*_c\mathcal{T}_S$, for all $c \in \mathcal{T}_S$.

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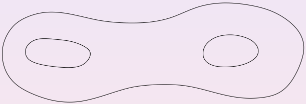
In the limit, the length of a curve is either 0 or a constant. Weighted multicurves are "limits" of some sequences of hyperbolic metrics, after scaling.



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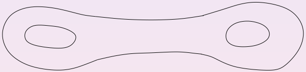

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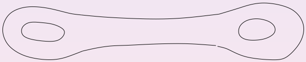

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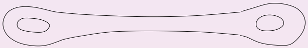
In the limit, the length of a curve is either 0 or a constant. Weighted multicurves are "limits" of some sequences of hyperbolic metrics, after scaling.



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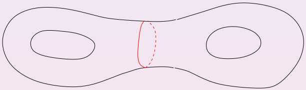

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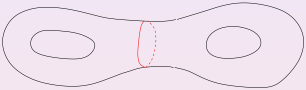

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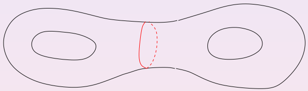

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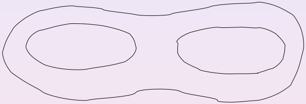
Fractional Dehn twists

Start with a hyperbolic surface.
Choose a simple closed geodesic c and $l > 0$, cut the surface open along it, rotate the right-hand side by l, then glue back.



This defines a homeomorphism $E^r_{c,l} : \mathcal{T}_S \to \mathcal{T}_S$.
If c' is another simple curve c', disjoint from c, then $E^r_{c,l}$ and $E^r_{c',l'}$ commute.
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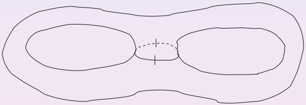

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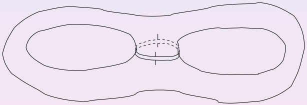

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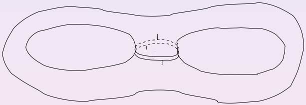

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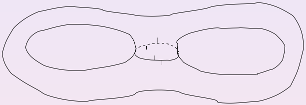

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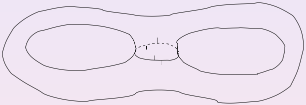

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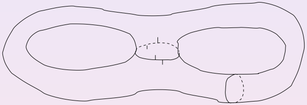

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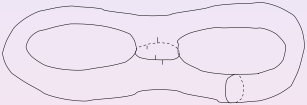

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The Earthquake Theorem

The action of weighted multicurves by fractional Dehn twist extends to :

$$E^r : \mathcal{ML}_S \times \mathcal{T}_S \to \mathcal{T}_S \ .$$

For $\lambda \in \mathcal{ML}_S$, $E^r(\lambda) : \mathcal{T}_S \to \mathcal{T}_S$ is a *right earthquake*. Correspondingly, left earthquakes : $E^l(\lambda) = E^r(\lambda)^{-1}$.

Thm (Thurston) : any $h, h' \in \mathcal{T}$ are connected by a unique right earthquake.

This provides another nice parametrization of $\mathcal{T}$ from $\mathcal{ML} \simeq T^*_h\mathcal{T}$, for a fixed $h \in \mathcal{T}$. Thurston sketched a proof, another (more analytic) was found by Kerckhoff.

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The 3-dim hyperbolic space can be defined as the hyperbolic plane. As a quadric in $\mathbb{R}^{3,1}$:

$$H^3 = \{x \in \mathbb{R}^4 \mid -x_0^2 + x_1^2 + x_2^2 + x_3^2 = -1 \& x_0 > 0\} \ .$$

There is a projective model, as the interior of the unit ball, and a Poincaré model, also in a ball (conformal). H^3 has a boundary at infinity, identified with $S^2 = \mathbb{C}P^1$.

Hyperbolic isometries act by complex projective transformations (Möbius transformation), in particular are complex. $Isom_+(H^3) = PSL(2, \mathbb{C})$.

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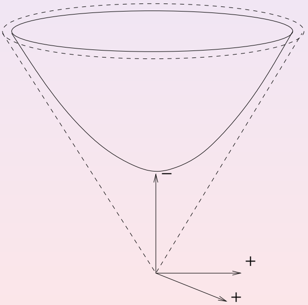

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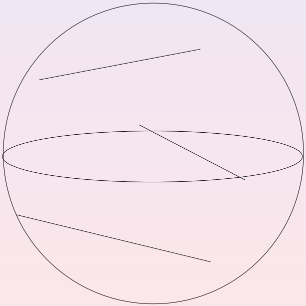

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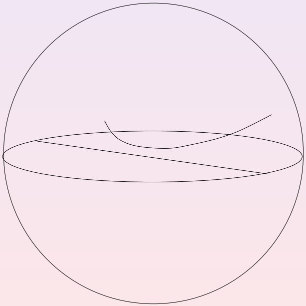

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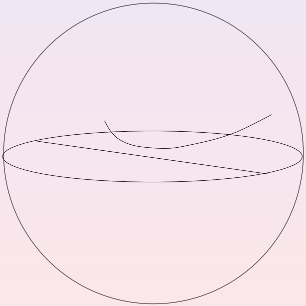

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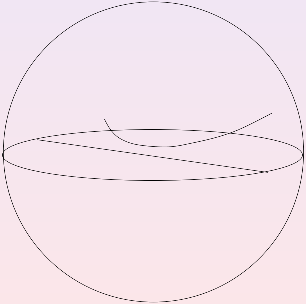

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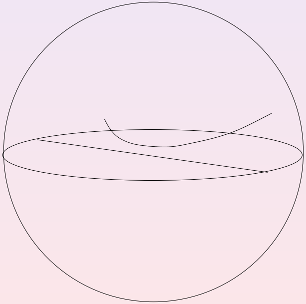

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For closed hyperbolic manifolds the situation is different than from hyperbolic surfaces.

Thm (Mostow, '70) : a closed 3-manifold admits at most one hyperbolic metric.

Moreover, those which do admit a hyperbolic metric are characterized in simple topological terms (Thurston '80, Perelman 2003) :

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Fuchsian manifolds

Start from a hyperbolic surface (S, g).

Consider $\rho : \pi_1(S) \to PSL(2, \mathbb{R})$, as acting on $H^2 \subset H^3$.
There is a unique extension as an action ρ' on H^3, from $PSL(2, \mathbb{R}) \subset PSL(2, \mathbb{C})$.
$H^3/\rho'(S) \simeq S \times \mathbb{R}$, metric : $dt^2 + \cosh^2(t) g$.

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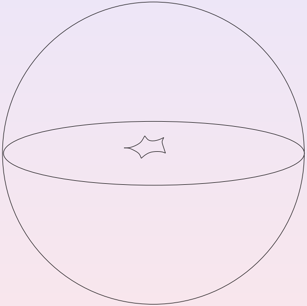

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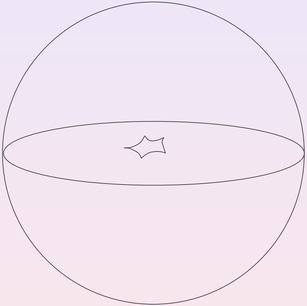

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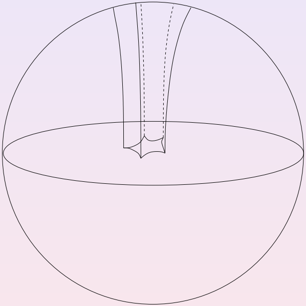

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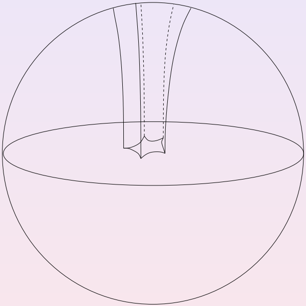

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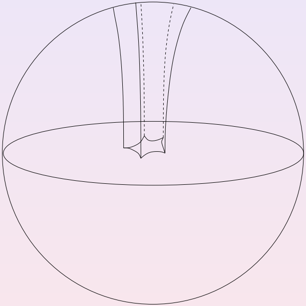

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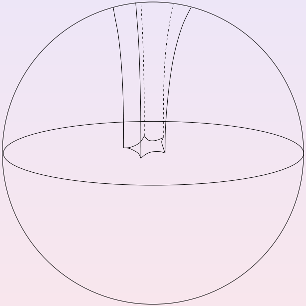

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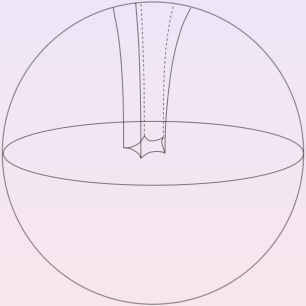

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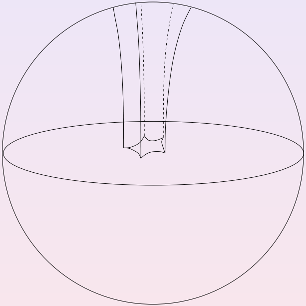

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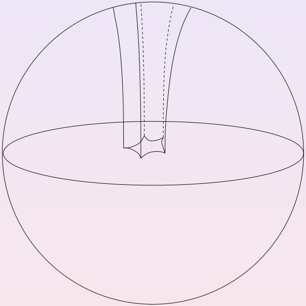

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Quasifuchsian manifolds

Def : a complete hyperbolic 3-mfld M is quasifuchsian if :

- $M \simeq S \times \mathbb{R}$, S closed of genus ≥ 2,
- M contains a compact convex subset K which is a deformation retract of M.

M has infinite volume and "grows" exponentially at infinity.

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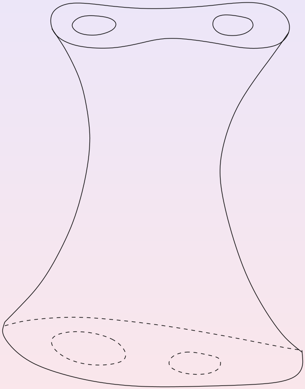

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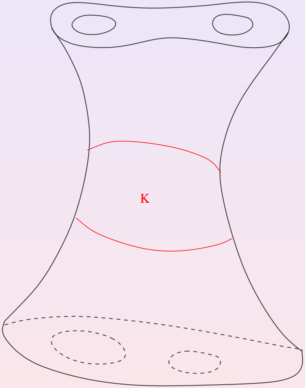

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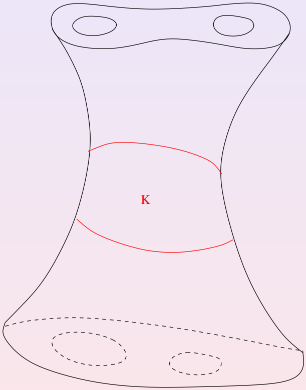

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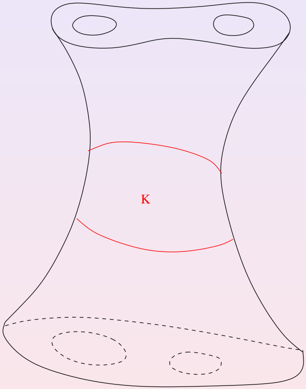

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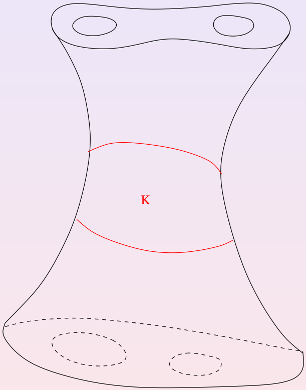

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