Hyperbolic geometry for 3d gravity

4. Quasifuchsian hyperbolic 3-manifolds

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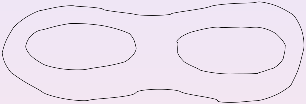
Fractional Dehn twists

Start with a hyperbolic surface.
Choose a simple closed geodesic c and $l > 0$, cut the surface open along it, rotate the right-hand side by l, then glue back.



This defines a homeomorphism $E^r_{c,l} : \mathcal{T}_S \to \mathcal{T}_S$.
If c' is another simple curve c', disjoint from c, then $E^r_{c,l}$ and $E^r_{c',l'}$ commute.
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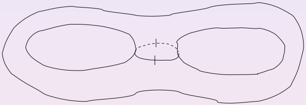

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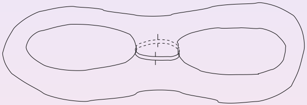

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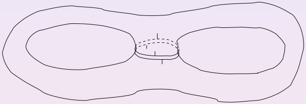

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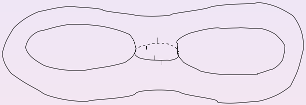

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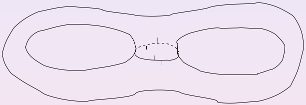

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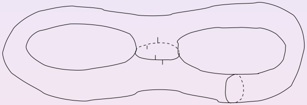

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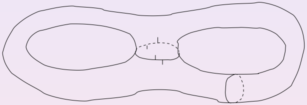

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The Earthquake Theorem

The action of weighted multicurves by fractional Dehn twist extends to :

$$E^r : \mathcal{ML}_S \times \mathcal{T}_S \to \mathcal{T}_S \ .$$

For $\lambda \in \mathcal{ML}_S$, $E^r(\lambda) : \mathcal{T}_S \to \mathcal{T}_S$ is a *right earthquake*. Correspondingly, left earthquakes : $E^l(\lambda) = E^r(\lambda)^{-1}$.

Thm (Thurston) : any $h, h' \in \mathcal{T}$ are connected by a unique right earthquake.

This provides another nice parametrization of $\mathcal{T}$ from $\mathcal{ML} \simeq T_h^*\mathcal{T}$, for a fixed $h \in \mathcal{T}$. Thurston sketched a proof, another (more analytic) was found by Kerckhoff.

In lecture 5 we will outline a simpler proof, based on AdS geometry (2+1D gravity).

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The 3-dim hyperbolic space can be defined as the hyperbolic plane. As a quadric in $\mathbb{R}^{3,1}$:

$$H^3 = \{x \in \mathbb{R}^4 \mid -x_0^2 + x_1^2 + x_2^2 + x_3^2 = -1 \& x_0 > 0\} .$$

There is a projective model, as the interior of the unit ball, and a Poincaré model, also in a ball (conformal). H^3 has a boundary at infinity, identified with $S^2 = \mathbb{C}P^1$.

Hyperbolic isometries act by complex projective transformations (Möbius transformation), in particular are complex. $Isom_+(H^3) = PSL(2, \mathbb{C})$.

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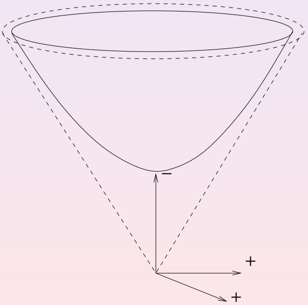

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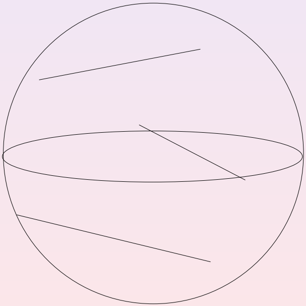

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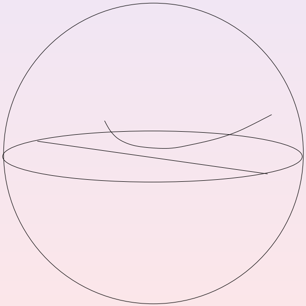

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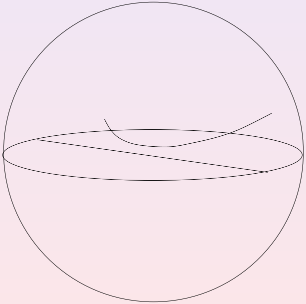

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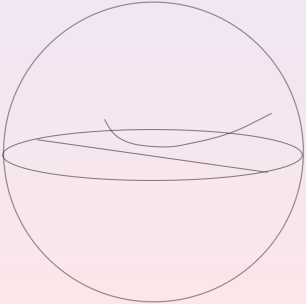

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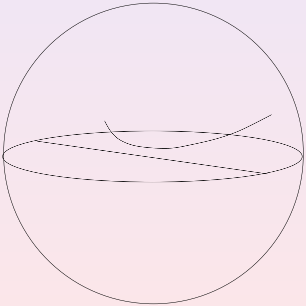

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Closed hyperbolic 3-manifolds

For closed hyperbolic manifolds the situation is different than from hyperbolic surfaces.

Thm (Mostow, '70) : a closed 3-manifold admits at most one hyperbolic metric.

Moreover, those which do admit a hyperbolic metric are characterized in simple topological terms (Thurston '80, Perelman 2003) :

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Fuchsian manifolds

Start from a hyperbolic surface (S, g).

Consider $\rho : \pi_1(S) \to PSL(2, \mathbb{R})$, as acting on $H^2 \subset H^3$.
There is a unique extension as an action ρ' on H^3, from $PSL(2, \mathbb{R}) \subset PSL(2, \mathbb{C})$.
$H^3/\rho'(S) \simeq S \times \mathbb{R}$, metric : $dt^2 + \cosh^2(t) g$.

Infinite volume, "grows" exponentially at infinity.
Let $\Lambda =$ limit set : accumulation points at infinity of the orbit of a point under $\rho'(\pi_1(S))$. Then Λ is the "equator".

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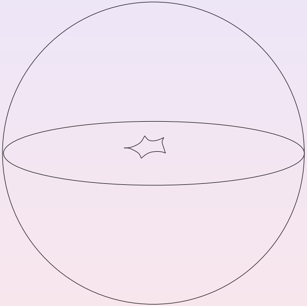

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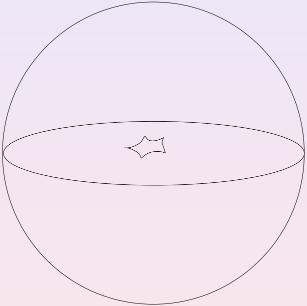

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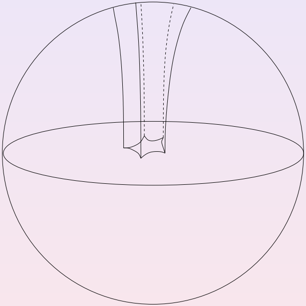

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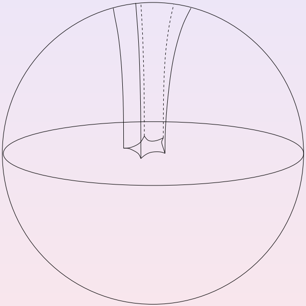

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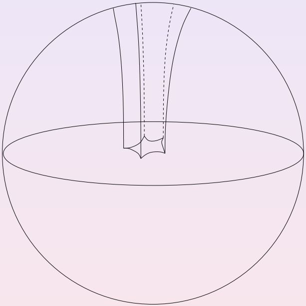

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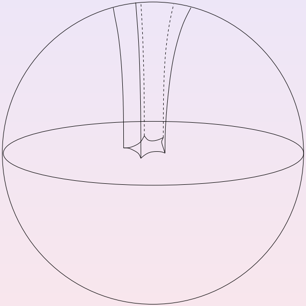

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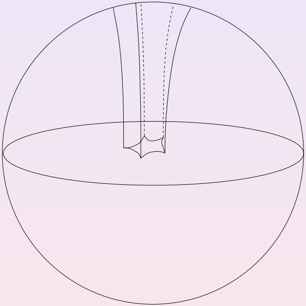

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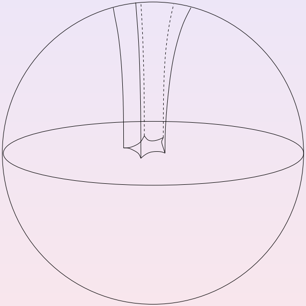

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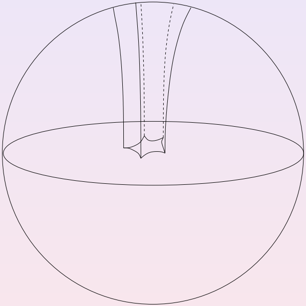

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Def : a complete hyperbolic 3-mfld M is quasifuchsian if :

- $M \simeq S \times \mathbb{R}$, S closed of genus ≥ 2,
- M contains a compact convex subset K which is a deformation retract of M.

M has infinite volume and "grows" exponentially at infinity.

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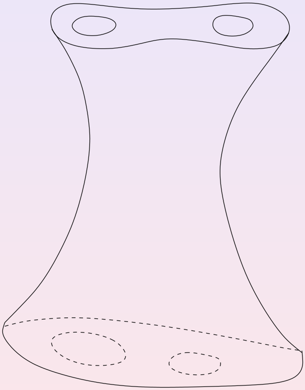

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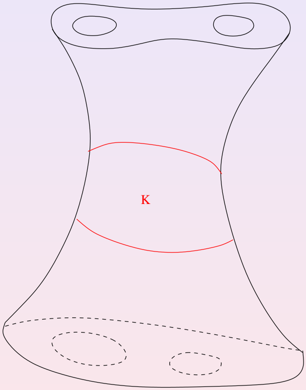

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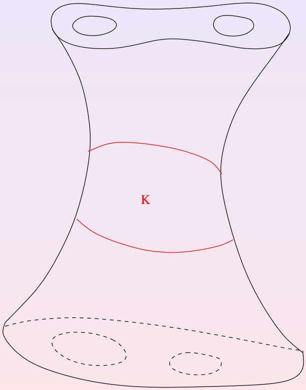

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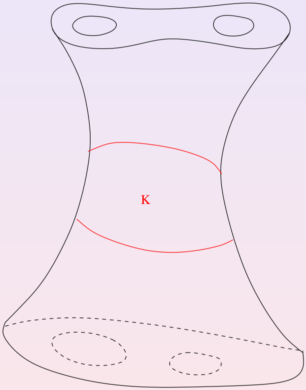

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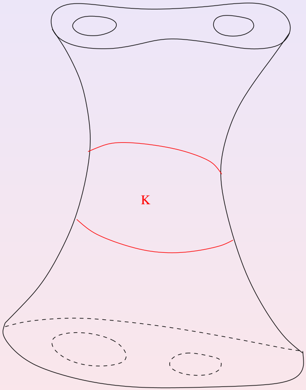

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The Bers double uniformization theorem

Let Λ be the limit set of $\rho : \pi_1(S) \rightarrow PSL(2, \mathbb{C})$, i.e. the accumulation points at infinity of the orbit of a point.

Lemma :

- Λ is a Jordan curve (it separates S^2 in two components Ω_+ and Ω_-) – strongly non-smooth.
- ρ acts properly discontinously on the complement of Λ.

The quotients $\Omega_\pm/\rho(\pi_1(S))$ correspond to the two boundary components of $M = S \times \mathbb{R}, \partial^\pm_\infty M$. Since ρ acts conformally on $S^2 = \partial_\infty H^3$, $\partial^\pm_\infty M$ have conformal structures $\tau_\pm$.

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Λ is invariant under $\rho(\pi_1(S))$, so it's convex hull $CH(\Lambda)$ is also invariant.

Lemma : $CC(M) = CH(\Lambda)/\rho(\pi_1(S)) \subset M$ is the smallest convex subset homotopically equivalent to M. It is called the convex core of M.

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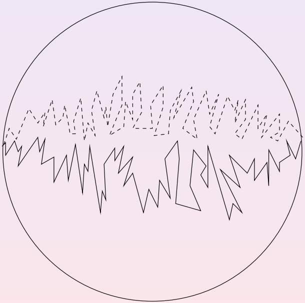

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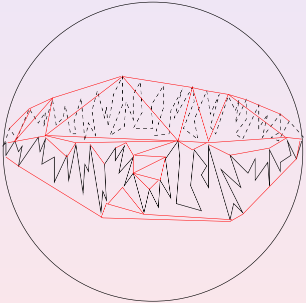

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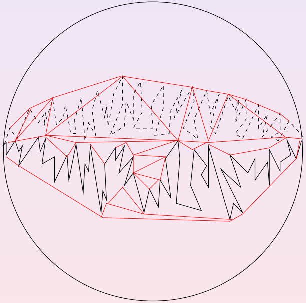

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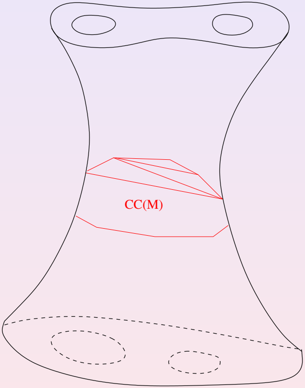

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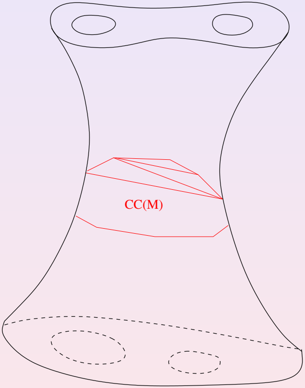

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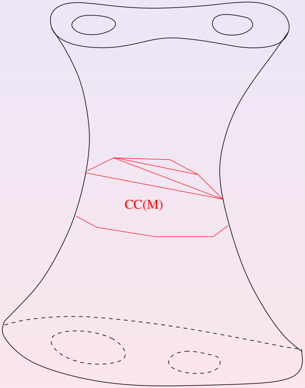

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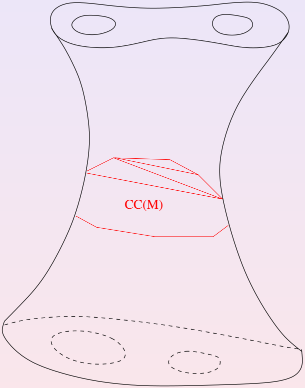

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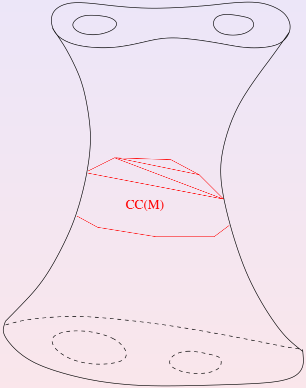

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The renormalized volume

Consider the equidistant surfaces $S_r^\pm$ from a compact subset K, with induced metric $I_r^\pm$. $e^{-2r}I^\pm \to I_\infty^\pm$. $[I_\infty^\pm] = \tau_\pm$, but $I_\infty^\pm$ depends on (S_r).
The volume between S_r^- andS_r^+ behaves as

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Thm (Takhtajan, Teo, 2002) : Under the constraint that $[I_\infty^\pm] = \tau_\pm$ and $I^\pm$ have fixed area, $V_R(I_\infty^-, I_\infty^+)$ is maximal iff $I_\infty^\pm$ have constant curvature.
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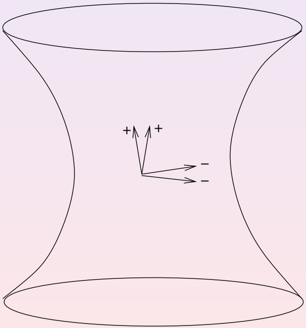
The AdS space

$$AdS^3 = \{x \in \mathbb{R}^{2,2} \mid -x_0^2 - x_1^2 + x_2^2 + x_3^2 = -1\} .$$

AdS^3 is a Lorentz space with constant curvature -1. It has a projective model (as for H^2), interior of a quadric Q.
$Isom_0(AdS^3) = PSL(2,\mathbb{R}) \times PSL(2,\mathbb{R})$: Q is ruled by two families of lines, preserved by $Isom_0(AdS^3)$.



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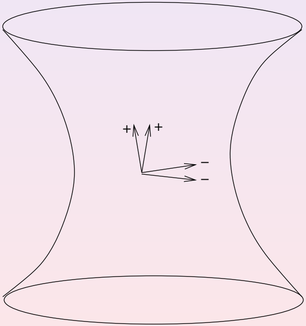

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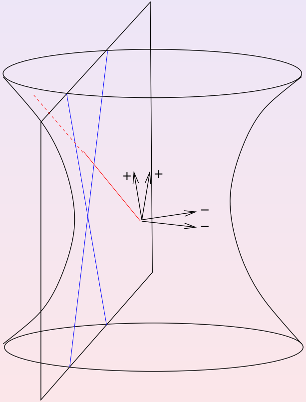

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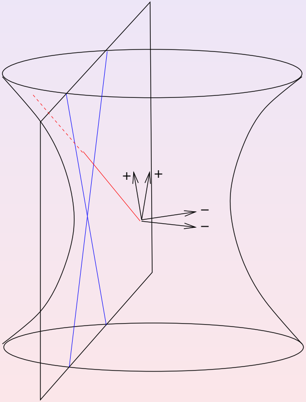

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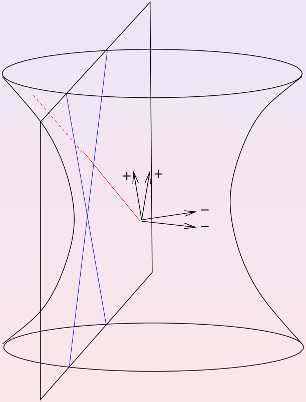

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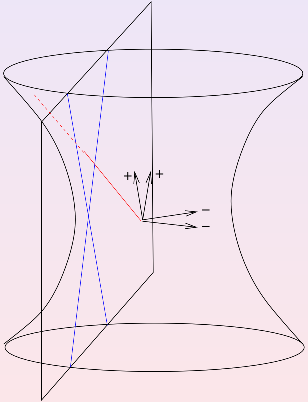

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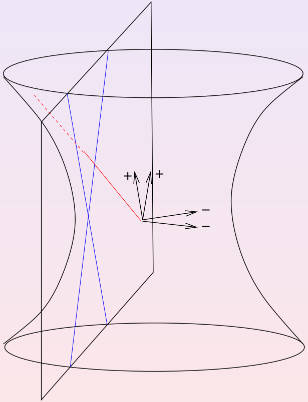

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GHMC AdS manifolds

An AdS 3-mfld is GHMC if :

- it is globally hyperbolic
- it contains a closed (oriented) space-like surface S of genus ≥ 2,
- it is maximal.

General idea : GHMC AdS mflds are very similar to quasifuchsian hyperbolic mflds.
Thm (Mess, 1990) : let M be a GHMC AdS mfld. Then $M = \Omega/\rho(\pi_1(S))$, where $\Omega \subset AdS^3$ is convex and $\rho : \pi_1(S) \rightarrow Isom_0(AdS^3)$. $\rho = (\rho_l, \rho_r) : \pi_1(S) \rightarrow PSL(2, \mathbb{R}) \times PSL(2, \mathbb{R})$, and $\rho_l, \rho_- \in \mathcal{T}_S$. Any $(\rho_l, \rho_r) \in \mathcal{T}_S$ can be uniquely obtained.
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