SUMMARY OF LETTERS TO PARTICIPANTS OF THE WORKSHOP
ON THE THEORY OF SHINICHI MOCHIZUKI,
DECEMBER 7–11 2015, OXFORD

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The webpage of the workshop on the inter-universal Teichmüller (IUT) theory, a.k.a. arithmetic deformation theory, of Shinichi Mochizuki, Oxford, December 7–11 2015 is
https://www.maths.nottingham.ac.uk/personal/ibf/files/symcor.iut.html

1. There will be 24–26 hours of lectures, for the list of speakers at the workshop see the workshop page. We will also arrange skype sessions of answers to them by Shinichi Mochizuki.

2. Questions from the participants of the workshop in relation to the theory: around 50 questions have been received and answered prior to the workshop. Answers to questions on frobenioids are available from http://www.kurims.kyoto-u.ac.jp/~motizuki/Responses%20on%20Frobenioids.pdf

3. Prerequisites for the workshop. They are mentioned or reviewed in sect. 1 of [17]: class field theory, especially local class field theory; elliptic curves over finite, local and global fields, some basic things about the set of Diophantine geometry conjectures the theory of Mochizuki deals with, basics of (classical) anabelian geometry. For the latter Ch.4 of [21] is a very easy to read source. A very short paper [1] elegantly proves a local version which involves higher ramification filtration, the result plays a role in IUT theory.

4. Introductory texts, surveys, slides of talks. The participants of the workshop are recommended to study the following materials, as well as the introductions of the main papers listed in section 7.

4i. Introductory texts by the author [13], [11], [12]. The first of these is the most recent text about certain analogies with Bogomolov’s proof of the geometric Szpiro inequality.

4ii. Notes [17] and a recent survey [19] provide an external perspective.

4iii. These slides of a talk by Yuichiro Hoshi at RIMS workshop on IUT, March 2015 [18] is a good introduction into mono-anabelian aspects of IUT. Slides of 3 talks by Yuichiro Hoshi at RIMS workshop in December 2015 are available from the top of this page http://www.kurims.kyoto-u.ac.jp/~yuichiro/talks_e.html. Slides and notes of talks at the December workshop are available from https://www.maths.nottingham.ac.uk/personal/ibf/files/iut-sch1.html.

5. Three motivating theories which can be read prior to the study of IUT, depending on your background and taste:

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○ the Bogomolov proof of the geometric version of the Szpiro inequality (sect. 5.3 of [14] and [22]), which involves geometric considerations that are substantially reminiscent of the geometry that underlies the Hodge theatres of [10]-I, for these analogies see 1.3, 2.10 of [17] and a very recent paper [13] by Shinichi Mochizuki. Citing Rk 2.3.4 of [10]-III, "Bogomolov’s proof may be thought of as a sort of useful elementary guide, or blueprint [perhaps even a sort of Rosetta stone!], for understanding substantial portions of the theory”.

○ the classical theory of the functional equation of the theta-function which was one important motivation for the development of the theory of [10]-II-III, see also [16] and [20] for more on the archimedean theta-function.

○ the classical theory of moduli of ordinary elliptic curves in positive characteristic and the related structure of the Hecke correspondence (i.e. $T_p$) in positive characteristic, which is also substantially reminiscent of the geometry that underlies the Hodge theatres of [10]-I.

6. Other analogies between IUT and other theories. In addition to class field theory, inverse Galois theory and anabelian geometry and the theories of the previous section, there are many other analogies.

There are analogies between IUT and $p$-adic Teichmüller theory and some analogies with complex Teichmüller theory which are well described in [11], [12] and in §I.4 of [10]-I.

There are certain analogies between IUT and $p$-adic Hodge theory (which is applied in the proofs of mono-anabelian geometry). For example, the local and global functoriality of absolute anabelian algorithms corresponds to some degree to compatible local isomorphisms between Galois cohomology modules in $p$-adic Hodge theory, see e.g. Fig.4.2 of [12].

Hodge–Arakelov theory [2] is not formally used in [10], but some of its ideas and expectations motivate key concepts and objects of [10]; for more on this see [11], [12]. IUT can be viewed as a mathematical justification and background for the realisation of a key idea from [2] concerning a possible approach to establishing the hyperbolic Vojta conjecture.

A number of aspects of the theory of [15] can be viewed as abelian ancestors of certain aspects of IUT, see Rk 2.3.3 of [10]-IV.

See the second part of sect. "Analogies and relations between IUT and other theories” of [17] for analogies with higher adelic geometry and analysis.

See also Rk. 3.3.2 of [10]-IV.

7. Main texts. The 4 main papers are

[10]-I-IV.

To read them, the following papers are also needed, to varying degree:

[9]-III, [7], [8], [3], [4], [6].

Concerning the last papers on frobenioids, there are certain expectations that most of their material could be omitted in future developments of the proper IUT theory. The theory of anabelioids is needed to understand the proof of Th. B in [10]-I.
8. Main theories and concepts. Below is the list of main theories and concepts. They are of different level of importance.

* mono-anabelian geometry, mono-anabelian reconstruction, [9], one mostly needs [9]-III; see also sect. 2.4 of [17]
* frobenoids [6], e.g. [6]-I, §6; tempered frobenoids are discussed in §3–§5 of [7]
* nonarchimedean theta-functions and related line bundles on tempered coverings, [7] plus comments to this paper on SM’s webpage, a review in §1 of [10]-II and a recently added Rk 2.3.4 of [10]-III; see also sect. 2.5 of [17]
* coric functions for and multiradical containers to transport elements of number fields, Rk 3.1.7 and Ex. 5.1 of [10]-I and Rks 1.5.2, 2.3.3, 3.12.2, Fig. 3.6, of [10]-III
* étale-like object and Frobenius-like objects, [9]-III
* generalised Kummer theory and multi-radial Kummer detachment [7], [10]-I-III; see also sect. 2.6 of [17]
* noncritical Belyi maps and their applications, [3], [8]
* Belyi cuspidalisation and elliptic cuspidalisation and its applications, [9]
* principle of Galois evaluation, [10]-II, applications of absolute anabelian geometry (in an extended form) to reconstruct a ring structure, [9]-III and [7]
* concept of mono-analiticity and arithmetic holomorphy immune to the logarithm, [9]-III
* concept of a global multiplicative subspace, [10]-I
* rigidities (discrete rigidity, constant multiple rigidity, cyclotomic rigidity) and indeterminacies, [7], [10]-II-III
* multiradiality and indeterminacies, [10]-II-III
* $\Theta^{\pm,\ell\ell}_{\text{NF theatres}}$, [10]-I-II-III
* theta-link and two types of symmetry, [10]-I-III; see also sect. 2.7 of [17]
* log-link, log-shell, log-Kummer correspondence and upper semi-compatibility, [9]-III, [10]-III
* log-theta-lattice, [10]-III

The key main theorems are Cor. 3.12 of [10]-III, Th. 1.10 and Cor. 2.2 of [10]-IV.


10. Other workshops and conferences on IUT

RIMS workshop on IUT, March 2015. The programme of this two week workshop included a 90 minutes talk by Yu. Hoshi and 67 1/2 hours of talks of G. Yamashita (thus, setting the world record). The slides for Hoshi’s talk, photos of formulas on blackboards from Yamashita’s lectures and a group photo are available from http://www.kurims.kyoto-u.ac.jp/~motizuki/research-english.html.

During the first week the following topics were covered:
○ various basic anabelian properties of absolute Galois groups of local fields that follow from local class field theory, such as cyclotomic rigidity (this constituted a substantial portion of Hoshi’s talk at the March workshop, as well as of the first few days of Yamashita’s talks, §1 of [5] and [1]
○ noncritical Belyi maps and their application in [8]
○ Belyi cuspidisation and its application to the theory of §1 of [9]-III
○ basic properties of theta functions on tempered coverings as discussed in §1–§2 of [7]
○ basic properties and examples of frobenioids, as in §6 of [6]-I
○ tempered frobenioids, i.e., a summary of §3–§5 of [7].

During the second week Yamashita’s talks essentially consisted of going through [10]-I-IV step by step, although with substantial simplifications; for instance, he only discussed the local theory in the case of bad reduction primes.

Beijing workshop on IUT, July 2015. This workshop on IUT, organised by Ch. P. Mok (Morningside Center of Math.) and F. Tan (Michigan State Univ.), was held in Beijing, Morningside Centre of Mathematics, July 20-25 2015: http://wiutt.csp.escience.cn/dct/page/1. Its programme is available from its pages. Four speakers K. Chen (Univ. Science and Technology of China), Ch. P. Mok, F. Tan, J. Tong (Univ. Bordeaux) delivered 24 hours of lectures. Several files related to the material of its lectures is available from the page of the workshop.

RIMS conference "IUT Summit" organised by Sh. Mochizuki (RIMS), Yu. Taguchi (Kyushu Univ.) and I. Fesenko will take place at RIMS, July 18-27 2016.

REFERENCES


[18] YU. HOSHI, Mono-anabelian reconstruction of number fields, a talk at the RIMS workshop on IUT, March 2015, available from http://www.kurims.kyoto-u.ac.jp/~yuichiro/talk20150309.pdf


