Comments on the G1BCOF Complex Functions Spring Semester Exam 2003-4

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1

(a) Generally well done. Not many people spotted the short cut that the integral of $2z$ round a closed curve is 0 by Cauchy's theorem.

(b) The answers here were somewhat inaccurate (especially the formula for the coefficients).

(c) This was mostly well done. However, it is important that the answers to (i) and (ii) should each be an explicit series, and not $(1+z^2)$ times a series: some people failed to multiply out. For the calculation of the integral, some people believed that the coefficient of $1/z$ in the series from (ii) was the same as the residue of the function at 0 (this is not true). This did not usually prevent them from obtaining the correct answer.

2

(a) The majority got this wrong! The Cauchy-Riemann equations should not come in to the definition of complex-differentiable, and the distinction between complex-differentiable and analytic needs to be made very clear. (See solutions!)

(b) This was fine.

(c) This was mostly well-done, though many people decided to expand $(x-y)^3$ (this is unnecessary), and often a mistake was made during the expansion (few people used the binomial theorem.) Only a small number of people used the quick method available involving Laplace's equation.

(d) There were a few slips in calculating the series, but this was mostly good.

3

(a) There were some problems with the precise definition of Log: remember that $\log(z)$ is not well-defined for a complex number (which is why we introduced the principal logarithm Log(z) in the first place).

(b) There were some difficulties in satisfying all the conditions given. Remember that you must have all three of z, w and zw in D_0 here. To get an example, you need to make sure that the principal arguments of z and w add up either to strictly more than π or strictly less than $-\pi$, in order for the jump at the negative real axis to play a role.

(c) There were many different errors made in these sketches. See solutions for the correct region.

(d) Those people who made it this far usually did quite well here, though some only found one solution.

(e) Very few people spotted the short cut here, involving anti-differentiating $1/z$. But most people who attempted this part got the right answer, or close.

4

The main problem here was with part (a). Part (b) was well-done, and part (c) was mostly correct (though some of the explanation tended to be unclear). With part (a) the main problem was in finding the singularities, which involved finding the 4th roots of -4. Many people appeared to have forgotten how to do this. Most people knew that the two square roots of -4 were $\pm 2i$, and this would lead to a solution if you know that the square roots of $\pm 2i$ are $\pm 1 \pm i$. Of course, you need to identify which of these singularities lie inside the contour (see solutions for further details).

5

(a) Most people who tried this question had problems solving $\mathrm{e}^z + 1 = 0$, with many saying that there were no solutions, even though everyone knew that $\mathrm{e}^{i\pi} = -1$ in Question 3!

(b) This was not well done. Although many people correctly set up some notation for the real and imaginary parts of f and g and stated the Cauchy-Riemann equations, most people made unwarranted assumptions about the form that f and g might have. In particular, very few people made convincing use of the chain rule for partial derivatives, which is the key to this question. See solutions for full details.