G1BCOF: Complex Functions

Problems for Special Examples Class

These problems are for the special examples class in teaching week 2 of Spring Semester. They combine some revision questions on complex numbers with a first introduction to the complex function $\exp(z)$.

Before this class you should read through Section 1.1 of the full lecture notes, which consists of revision of material covered in G11LMA. For the module G1BCOF you are expected to make sure that you are very familiar with this basic material on complex numbers.

Try to solve the problems before the class. Solutions to the revision questions will be discussed after 15 minutes of the class. You will also have an opportunity at this point to ask questions on this basic material.

1 Revision questions

Problem 1.1 Find all the complex solutions to the following equation.

$$z^4 + 16 = 0 \ :$$

(i) in exponential form; (ii) in the form $x + yi$.
[Hint: rewrite this as $z^4 = -16$.]

Problem 1.2 Find the two complex solutions to the equation

$$z^2 + (3 + 3i)z + 4i = 0.$$

[Hint: first find the two complex square roots of $2i$.]

Please turn over for questions on the exponential function

2 The complex exponential function

Given a complex number $z = x + iy$, we already know how to define $\exp(x) = \mathrm{e}^x$ (a real number) and $\exp(iy) = \mathrm{e}^{iy}$ (a complex number of modulus 1). It is standard to define

$$\exp(z) = \exp(x)\exp(iy) = \mathrm{e}^x \mathrm{e}^{iy}.$$

We may also write $\mathrm{e}^z = \exp(z)$.

Problem 2.1 (a) Let z be a complex number. Show that $|\exp(z)| = \exp(\mathrm{Re}(z))$.

(b) Now suppose that z and w are complex numbers with $\exp(z) = w$. Show that $w \neq 0$, $\mathrm{Re}(z) = \ln(|w|)$ and that $\mathrm{Im}(z)$ is an argument of w.

Problem 2.2 For each of the following equations, find all the complex solutions (if any): (i) $\exp(z) = 1 + i$; (ii) $\mathrm{e}^z + 1 = 0$.

Problem 2.3 Let w be a non-zero complex number. Show that there are infinitely many different complex numbers z solving the equation

$$\exp(z) = w.$$

How are these different solutions related?

The exponential function and complex logarithms are discussed in more detail in Section 2.5 of the full lecture notes.