

Optimization of Quantum Information Processing at Mutual Information Criterion*

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Abstract

A model of quantum noisy channel with input encoding by a classical random vector is described. An equation of optimality is derived to determine a complete set of wave functions describing quantum decodings based on quasi-measurements maximizing the classical amount of transmitted information. A solution of this equation is found for the Gaussian multimode case with input Gaussian distribution. It is described by the overcomplete family of coherent vectors describing an optimal quasimeasurement of the canonical annihilation amplitudes in the output Hilbert space. It is found that the optimal decoding in this case realizes the maximum amount $I = \text{Sp} \ln [1 + S/(N+1)]$ of the classical information as transmitted via the classical Gaussian channel with the effective noise covariance matrix $N + I$. A physical realization of optimal quasi-measurement based on an indirect (heterodyne) observation of the canonical operators is suggested.

Introduction

The use of homodyne or phase-sensitive receivers for the reception of a coherently modulated quantum signal enables one to extract in the quasiclassical limit only a half of the encoded information. The coherent reception of quantum signals, based on simultaneous direct measurement of amplitude and phase (or phase coordinates of each mode of the signal), is forbidden due to the uncertainty relations of quantum theory. Therefore, it is of interest to investigate the potential possibilities of decoding in quantum communication channels based on the indirect, heterodyne methods of observation. The theory of indirect

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measurements of incompatible observables $\{b_j\}$ has been under the development starting from the pioneering studies [3, 4, 7] to meet the demands of the quantum communication theory. Using a quantum reformulation of the statistical inference theory for a Bayesian risk criteria it was recently shown how to formulate optimal quantum information processings based on the indirect measurements in the classical-quantum noisy channels.

The use of the concept of indirect measurement has made it possible to indicate for a Gaussian classical-quantum communication channel the amount of information

$$I = \ln \left(1 + \frac{s}{n+1} \right),$$

transmitted by each mode of coherently modulated signal in using a quantum receiver with a large linear amplifier* (s, n are the energies of the corresponding modes of the classical input signal and of the quantum additive noise expressed in units of $h\nu$). The same amount of information was deduced in [5], where a method of its decoding was indicated, leading to indirect measurement. The rate of information transmission by the coherent signal was also determined in an earlier work [6], where vectors of the $\{\varphi_\beta\}$ coherent states were used as the operators characterizing the coherent reception. The problem of the realization of the coherent measurement and its optimality was not considered. However the natural question – whether the decoding based on the quasi-measurement described by the coherent vectors (1.4) is optimal, and if so, in what conditions it is unique, has not been resolved.

In order to answer this question we derive the equation determining the optimal decoding vectors $\{\varphi_\beta\}$ according to the criterion of the maximum amount of information and specify the optimal indirect measurement realizing the corresponding quasi-measurement.

1 Decodings based on quasi-measurements

Quasi-measurement of noncommuting operators $\{b_j\}$ with values in a given (measurable) space $B \ni \beta$ are described by operator-valued (positive, σ -additive) measures $\Pi(d\beta)$ normalized to the identity operator $\hat{1}$ in the Hilbert space \mathbb{H} of quantum states as $\Pi(B) = \hat{1}$ such that

$$\hat{1} = \int \Pi(d\beta), \quad \int \beta_j \Pi(d\beta) = b_j. \quad (1.1)$$

If the resolution of identity in (1.1) is orthogonal such that $\Pi(d\beta)\Pi(d\beta') = \Pi(d\beta \cap d\beta')$, then it describes the usual (direct) measurement of compatible observables in the spectral decomposition $b_j = \int \beta_j \Pi(d\beta)$. In the general case, however, the operators $\{b_j\}$ do not have common orthogonal spectral decomposition, and it can be said that the nonorthogonal expansion (1.1) describes their

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simultaneous quasi-measurement. The probabilistic operator measure $\Pi(d\beta)$ is an analog of classical randomized decision rules $P(d\beta|b)$.

The classical probability distribution $P(d\beta)$ of the results of quasi-measurements in the state specified by the statistical density operator ρ is determined by the Hilbert space trace Tr of its product with the quantum probability measure $\Pi(d\beta)$ in the same way as in the case of direct measurements:

$$P(d\beta) = \text{Tr} \Pi(d\beta)\rho. \quad (1.2)$$

Below we shall specify the operator $\Pi(d\beta)$ in the form

$$\Pi(d\beta) = \varphi_\beta \varphi_\beta^* \mu(d\beta), \quad (1.3)$$

explicitly taking account of its positive definiteness and σ -additiveness. Here $\{\varphi_\beta, \beta \in B\}$ is a family of non-Hermitian operators acting in general from a Hilbert space \mathbb{F} into the original space \mathbb{H} and, together with a positive measure $\mu(d\beta)$ as the reference measure on the set B , satisfying the condition of normalization (1.1) as the completeness condition for $\{\varphi_\beta\}$. In particular, if $\{\varphi_\beta\}$ are operators acting from one-dimensional complex space $\mathbb{F} = \mathbb{C}$ into \mathbb{H} , i.e., are a set of wave functions $\varphi_\beta \in \mathbb{H}$ (in general nonorthogonal and non-normalized: $\varphi_\beta^* \varphi_{\beta'} \neq \delta_{\beta\beta'}$), then the quasi-measurement specified by them is called elementary. We note that any quasi-measurement can be realized by direct measurement of a certain family of compatible observables $\{\hat{\beta}_j\}$ in an expanded quantum system consisting of the original system and an independent auxiliary system. This follows from the well-known fact that any operator measure $\Pi(d\beta)$ can be represented as partial averaging $\Pi(d\beta) = \text{Tr}_{\mathbb{H}_0} \Theta(d\beta) \rho_0$ of an orthoprojector measure $\Theta(d\beta)$ given by common eigenvectors of the commuting operators $\hat{\beta}_j = \int \beta_j \Theta(d\beta)$ acting in the expanded space $\mathbb{H} \otimes \mathbb{H}_0$, along with the state ρ_0 of the auxiliary system described by a Hilbert space \mathbb{H}_0 . Such a realization of quasi-measurement is called *indirect measurement*.

As an example we consider an elementary quasi-measurement specified by the coherent vectors

$$\varphi_\alpha = \exp \left\{ -\frac{1}{2} \alpha^\dagger \alpha + a^\dagger \alpha \right\} |0\rangle = |\alpha\rangle. \quad (1.4)$$

It can be realized by direct simultaneous measurement of the complex amplitudes

$$\hat{\alpha} = a + a_0^+ = (\hat{\alpha}_j), \quad j = 1, \dots, r, \quad (1.5)$$

where $\alpha = (\alpha_j)$ and $a = (a_j)$ are columns of the amplitudes $\alpha_j \in \mathbb{C}$ and the annihilation operators a_j operating into the original Hilbert space \mathbb{H} , α^\dagger and a^\dagger are the adjoint rows, while a_0^+ denotes the column (a_{0j}^+) of creation operators a_{0j}^+ of an auxiliary (heterodyning) system in the vacuum state $|0\rangle_0 \in \mathbb{H}_0$. For this purpose it is sufficient to note that the partial averaging over the vacuum state $|0\rangle_0$ amounts to the determination of the projections φ_α of the eigenvectors

$\psi_\alpha \in \mathbb{H} \otimes \mathbb{H}_0$ of commuting operators $\alpha = a + a_0^+$ in the original space \mathbb{H} . One can easily verify that the generalized vectors

$$\psi_\alpha = \exp \left\{ - (a - \alpha)^\dagger a_0^+ \right\} |0\rangle_0 \otimes |\alpha\rangle \quad (1.6)$$

satisfy the equations $\hat{\alpha}\psi_\alpha = \alpha\psi_\alpha$, $\hat{\alpha}^*\psi_\alpha = \alpha^*\psi_\alpha$ and form a complete orthonormal set with the measure

$$d\mu(\alpha) = \prod_{j=1}^r \frac{1}{\pi} d \operatorname{Re} \alpha_j d \operatorname{Im} \alpha_j.$$

Multiplying these vectors from the left by ${}_0\langle 0|$ we find that the projections $\varphi_\alpha \in \mathbb{H}$ are actually the coherent vectors $|\alpha\rangle$ and, together with the measure $\mu(d\alpha) = d\mu(\alpha)$, determine ideal measurement corresponding to the indirect measurement of noncommuting a_j .

The question – whether the decoding based on the quasi-measurement described by the coherent vectors (1.4) is optimum and if so in what conditions – is of interest. In order to answer this question we are going to derive the equation determining the operators φ_β , which are optimal according to the criterion of the maximum amount of mutual information and specify the optimum quasi-measurement.

We shall assume that the family of statistical operators $\{\rho(\vartheta)\}$ defining the state of the quantum communication channel is given as a function of the random information parameters $\vartheta = (\vartheta_j)$ having the distribution $P(d\vartheta)$. The amount of Shannon information of the parameters ϑ and the results of quasi-measurements β is determined by the usual formula

$$I_{\beta, \vartheta} = \iint p(\beta|\vartheta) \ln \frac{p(\beta|\vartheta)}{p(\beta)} P(d\vartheta) \mu(d\beta),$$

where according to (1.2), (1.3) the density $p(\beta|\vartheta)$ has the form

$$p(\beta|\vartheta) = \operatorname{Tr} \varphi_\beta \varphi_\beta^* \rho(\vartheta) = \operatorname{Tr} \varphi_\beta^* \rho(\vartheta) \varphi_\beta, \quad (1.7)$$

$p(\beta) = \int p(\beta|\vartheta) P(d\vartheta)$, satisfying the normalization condition

$$\int p(\beta|\vartheta) \mu(d\beta) = 1, \quad \int p(\beta) \mu(d\beta) = 1.$$

Using the method of Lagrangian multipliers we set up a function

$$\int \left(\int i(\beta, \vartheta) \operatorname{Tr} \varphi_\beta^* \rho(\vartheta) \varphi_\beta P(d\vartheta) - \operatorname{Tr} \varphi_\beta^* \hat{\lambda} \varphi_\beta \right) \mu(d\beta),$$

where $i(\beta, \vartheta) = \ln [p(\beta|\vartheta) / \int p(\beta|\vartheta) P(d\vartheta)]$, and $\hat{\lambda}$ is an operator determined from the condition of completeness $\int \varphi_\beta \varphi_\beta^* \mu(d\beta) = \hat{1}$. Varying it over φ_β^* we obtain the equation determining optimum φ_β :

$$\left(I(\beta) - \hat{\lambda} \right) \varphi_\beta = 0, \quad (1.8)$$

where $I(\beta) = \int i(\beta, \vartheta) \rho(\vartheta) P(d\vartheta)$ and $\hat{\lambda} = \int I(\beta) \varphi_\beta \varphi_\beta^* \mu(d\beta)$.

In the derivation of this equation we have made use of the fact that

$$\int p(\beta|\vartheta) \delta i(\beta, \vartheta) (\beta, \vartheta) P(d\vartheta) = \int p(\beta|\vartheta) \delta \varphi^* \left(\frac{\rho(\vartheta)}{p(\beta|\vartheta)} - \frac{\rho}{p(\beta)} \right) \varphi_\beta P(d\vartheta) = 0,$$

where $\rho = \int \rho(\vartheta) P(d\vartheta)$. Multiplying equation (1.8) from the right by φ_β^* and integrating with the measure $\mu(d\beta)$ we determine the operator $\hat{\lambda}$:

$$\hat{\lambda} = \iint \rho(\vartheta) i(\beta, \vartheta) \varphi_\beta \varphi_\beta^* \mu(d\beta) P(d\vartheta),$$

the trace of which $\text{Tr } \hat{\lambda}$ gives the maximum amount of information. And, finally, eliminating the operator from Equation. (1.8) and multiplying it from the left by $\hat{\lambda}$, we rewrite this equation in the following equivalent (because of the condition of completeness $\int \varphi_\alpha \varphi_\alpha^* \mu(d\alpha) = \hat{1}$) form:

$$\int \varphi_\alpha^* \rho(\vartheta) \left(\varphi_\beta i(\beta, \vartheta) - \int \varphi_{\beta'} \varphi_{\beta'}^* \varphi_\beta i(\beta', \vartheta) u(d\beta') \right) P(d\vartheta) = 0. \quad (1.9)$$

The equation thus obtained is a complex nonlinear equation in φ_β and it is not possible to solve it explicitly in the general case. However, as will be shown in the next section, in the Gaussian case the coherent vectors (1.4) are optimum operators φ_β .

2 Optimal decoding in quantum Gaussian channel

Let the received signal b be a superposition $b = \vartheta + a$ of a complex vector $\vartheta = (\vartheta_j, j = 1, \dots, r)$ and an independent Gaussian Boson noise $a = (a_j)$ (a_j are annihilation operators of the investigated modes: $[a_j, a_k^\dagger] = \delta_{jk}$, having zero mathematical expectation $\langle a_j \rangle = 0$ and specified average number of quanta $\langle a_j^\dagger a_j \rangle = n_j$). In Glauber's representation [2] the statistical operator of this noise has the form

$$\rho = \int |\alpha\rangle \langle \alpha| |N|^{-1} \exp\{-\alpha^\dagger N^{-1} \alpha\} d\mu(\alpha), \quad (2.1)$$

where $|\alpha\rangle$ is the coherent vectors (1.4); N is a matrix whose eigenvalues are n_j , and $|N| = \det N = n_1 \cdots n_r$. In formula (2.1) the matrix form of writing scalar products is used, $\alpha^\dagger N^{-1} \alpha = \sum_{j,k} \alpha_j^* (N^{-1})_{jk} \alpha_k$, similar as $a^\dagger \alpha = \sum_j a_j^\dagger \alpha_j$ in (1.4). The signal b after passing through the indicated linear channel is described by a family of operators $\{\rho(\vartheta)\}$ of the form

$$\rho(\vartheta) = \int |\alpha\rangle \langle \alpha| p(\alpha|\vartheta) d\mu(\alpha), \quad (2.2)$$

where $p(\alpha|\vartheta) = |N|^{-1} \exp\{-(\alpha-\vartheta)^\dagger N^{-1}(\alpha-\vartheta)\}$. The probability distribution $P(d\vartheta)$ of the transmitted signal ϑ , chosen from the condition of maximum entropy and finiteness of the energy $\langle \vartheta^\dagger S^{-1} \vartheta \rangle \leq (S$ is the given correlation matrix $\langle \vartheta_j^* \vartheta_k \rangle)$, also has a Gaussian form:

$$P(d\vartheta) = |S|^{-1} \exp\{-\vartheta^\dagger S^{-1} \vartheta\} d\mu(\vartheta), \quad d\mu(\vartheta) = \prod_{j=1}^r \frac{1}{\pi} d\text{Re } \vartheta_j d\text{Im } \vartheta_j. \quad (2.3)$$

In the case under investigation it is natural to expect that the results of optimum quasi-measurement of the Gaussian observable b also have a Gaussian distribution; among the Gaussian operators φ_β , determining this distribution, it is natural to separate out the operators characterized by the minimum uncertainty relation $\langle (b-\beta)_j^\dagger (b-\beta)_k \rangle = \delta_{jk}$. Such operators are coherent vectors.

Let us verify if vectors φ_β of form (1.4) satisfy Equation. (1.9) in the Gaussian case. Substituting (1.4), (2.2) into (1.7) and taking into account that

$$\langle \beta | \alpha \rangle = \exp\{\beta^\dagger \alpha - (\alpha^\dagger \alpha + \beta^\dagger \beta)/2\}$$

we can find the conditional density

$$\begin{aligned} p(\beta|\vartheta) &= \int \langle \beta | \alpha \rangle \langle \alpha | \beta \rangle p(\alpha|\vartheta) d\mu(\alpha) \\ &= |N + I|^{-1} \exp\{- (\beta - \vartheta)^\dagger (N + I)^{-1} (\beta - \vartheta)\}, \end{aligned} \quad (2.4)$$

and also the function $i(\beta, \vartheta)$ occurring in Equation. (1.9):

$$i(\beta, \vartheta) = \ln |I + S(N + I)^{-1}| + \vartheta^\dagger S^{-1} \vartheta - (\vartheta - A\beta)^\dagger G(\vartheta - A\beta), \quad (2.5)$$

where $A = S(S + N + I)^{-1}$, and $G = S^{-1} + (N + I)^{-1}$. In (2.5) only the last term is significant for Equation. (1.9): the first two terms drop out after substitution into this equation. Substituting the last term $(A\beta - \vartheta)^\dagger G(A\beta - \vartheta) \equiv c(A\beta, \vartheta)$ of function (2.5) into Equation. (1.9) and using representation (2.2) of the operator $\rho(\vartheta)$, we verify that the coherent vector $\varphi_\beta = |\beta\rangle$ is a solution of Equation. (1.9). Thus the solution of the formulated problem is reduced, as also in the optimization according to the mean square criterion, to the verification of the identity

$$\begin{aligned} &\iint \langle \alpha | \alpha' \rangle \langle \alpha' | \beta \rangle \left(c(A\beta, \vartheta) - \int \frac{\langle \alpha' | \beta' \rangle \langle \beta' | \beta \rangle}{\langle \alpha' | \beta \rangle} c(A\beta', \vartheta) d\mu(\beta') \right) \\ &\times p(\alpha'|\vartheta) d\mu(\alpha') P(d\vartheta) = 0, \end{aligned} \quad (2.6)$$

where $p(\alpha|\vartheta)$, $P(d\vartheta)$ are defined in (2.2), (2.3).

Below we shall require the following formulas of integration in the complex

r -dimensional space $\mathbb{C}^r \ni z$:

$$\begin{aligned}
\int \exp \{ - (z - \alpha)^\dagger Q (z - \beta) \} |Q| d\mu(z) &= 1, \\
\int z \exp \{ - (z - \alpha)^\dagger Q (z - \beta) \} |Q| d\mu(z) &= \beta, \\
\int z^* \exp \{ - (z - \alpha)^\dagger Q (z - \beta) \} |Q| d\mu(z) &= \alpha^*, \\
\int (z - \alpha)^\dagger H (z - \beta) \exp \{ - (z - \alpha)^\dagger Q (z - \beta) \} |Q| d\mu(z) &= \text{Sp } Q^{-1} H,
\end{aligned} \tag{2.7}$$

all positive-definite $(r \times r)$ -matrices Q, H , where Sp is the trace in the space \mathbb{C}^r where all vectors $\alpha, \beta \in \mathbb{C}^r$ are taking values. Taking into account that

$$\langle \alpha' | \beta \rangle \langle \beta' | \beta \rangle (\langle \alpha' | \beta \rangle)^{-1} = \exp \{ - (\beta' - \alpha')^\dagger (\beta' - \beta) \},$$

in the identity (2.6) we can carry out the integration over z after making the substitution $A\beta' = z$ and making use of formula (2.7) for $Q = (AA^\dagger)^{-1}, H = G$. As a result the expression in the parentheses in (2.6) becomes $(\beta - \alpha')^\dagger A^\dagger G (A\beta - \vartheta) \text{Sp } AA^\dagger G$. The integration of the last expression over α' with the density

$$\begin{aligned}
&\langle \alpha | \alpha' \rangle \langle \alpha' | \beta \rangle p(\alpha' | \vartheta) \\
&= \langle \alpha | \beta \rangle |N + I|^{-1} |\bar{Q}| \exp \{ - (\vartheta - \alpha)^\dagger (N + I)^{-1} (\vartheta - \beta) - (\alpha' - \bar{\alpha})^\dagger \bar{Q} (\alpha' - \beta) \}
\end{aligned}$$

amounts to the use of the first and the third formulas of integration (2.7), where for Q, α, β we should take $\bar{Q} \equiv N^{-1} + I, \bar{\alpha} \equiv (N + I)^{-1} (\vartheta + N\alpha), \beta \equiv (N + I)^{-1} (\vartheta + N\beta)$. This gives

$$\begin{aligned}
&\left[(\beta - (N + I)^{-1} (\vartheta + N\alpha))^\dagger A^\dagger G (A\beta - \vartheta) - \text{Sp } AA^\dagger G \right] \langle \alpha | \beta \rangle \\
&\times |N + I|^{-1} \exp \{ - (\vartheta - \alpha)^\dagger (N + I)^{-1} (\vartheta - \beta) \}.
\end{aligned} \tag{2.9}$$

In order to complete the verification of identity (2.6) it only remains to carry out averaging with the distribution $P(d\vartheta)$. Rewriting the expression in the square brackets in (2.9) in the form

$$(\vartheta - (N + I)\beta + N\alpha)^\dagger (N + I)^{-1} A^\dagger G (\vartheta - A\beta) - \text{Sp } AA^\dagger G$$

and integrating it with respect to ϑ with the density

$$\begin{aligned}
&|N + I|^{-1} |S|^{-1} \exp \{ - (\vartheta - \alpha)^\dagger (N + I)^{-1} (\vartheta - \beta) - \vartheta^\dagger S^{-1} \vartheta \} \\
&= |G| |S + N + I| e^{-(\vartheta - A\alpha)^\dagger G (\vartheta - A\beta)}
\end{aligned}$$

according to the formulas (2.7) for $Q = G, H = (N + I)^{-1} A^\dagger G$ we obtain

$$|S + N + I|^{-1} \text{Sp} (G^{-1} (N + I)^{-1} A^\dagger - AA^\dagger) G.$$

This expression coincides with the left-hand side of (2.6) with an accuracy up to a nonzero factor $\langle \alpha | \beta \rangle$. Due to the identity

$$G^{-1} = (S^{-1}(N + I)^{-1})^{-1} = S(S + N + I)^{-1}(N + I) = A(N + I),$$

as a consequence of which

$$\text{Sp} (G^{-1}(N + I)^{-1}A^\dagger - AA^\dagger)G = \text{Sp} (AA^\dagger - AA^\dagger)G = 0,$$

the validity of the verified equation is follows straightforward.

Thus in a linear Gaussian Bosonic channel the optimum decoding is given by coherent vectors of the form (1.4). The ideal quasi-measurement, defined by the overcomplete nonorthogonal set $\{|\beta\rangle\}$ of coherent vectors, is realized by the measurement of complex amplitudes

$$\hat{\beta} = b + a_0^\dagger = \vartheta + a + a_0^\dagger = \vartheta + \alpha, \quad (2.10)$$

in the expanded space these amplitudes are of a complete orthonormal set of eigenvectors. The measurement of simultaneous observables (2.10) is a linear indirect measurement of noncommuting $b = (b_j)$ investigated in [1]. The maximum amount of information decoded by the optimal ideal quasi-measurement is easily found by averaging the function (2.5):

$$\mathsf{I} = \ln |I + S(N + I)^{-1}| = \text{Sp} \ln (I + S(N + I)^{-1}).$$

In the one-dimensional case the obtained amount of information coincides with that shown in [5, 6].

In conclusion, we note that if a Gaussian stationary signal transmitted along the channel is mixed with stable thermal noise of temperature $\theta = kT$, then the rate information transmission in optimum decoding is determined by the information

$$\ln (1 + s_\nu / (n_\nu + 1))$$

transmitted by each mode summed over the operating frequency range $N \ni \nu$:

$$\mathsf{I} = \int_N \ln (1 + s_\nu (n_\nu + 1)^{-1}) d\nu = \int_N (1 + s_\nu (1 - e^{-h\nu/\theta})) d\nu. \quad (2.11)$$

Here s_ν is the spectral intensity of the transmitted signal expressed in units of $h\nu$, and $n_\nu = (e^{h\nu/\theta} - 1)^{-1}$ is the spectral intensity of the noise. In the classical limit $h\nu/\theta \ll 1$ the rate of information transmission goes over into the corresponding classical expression $\mathsf{I} = \int \ln(1 + \sigma_\nu^2/\theta) d\nu$, where $\sigma_\nu^2 = s_\nu h\nu$. In the opposite case $h\nu/\theta \gg 1$ of low temperatures the transmission rate $\mathsf{I} = \int \ln(1 + s_\nu) d\nu$ remains finite in contrast to the classical case. It is not difficult to see that the quantum corrections to (2.11) are significant only for weak (at the receiving end of the channel) signals $s_\nu \ll 1$, for which formula (2.11) can be written in the form $\mathsf{I} \approx \int (1 - e^{-h\nu/\theta}) s_\nu d\nu$.

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