Problem class 9

1. Consider the curve $y^2 = (x+4)(x+1)(x-5)$ defined over $\mathbb{Q}$. We find a point $P = (-3, 4)$ of infinite order in $E(\mathbb{Q})$. Show that $E(\mathbb{Q})$ has rank 1.

2. Let $E : y^2 = (x - e_1)(x - e_2)(x - e_3)$ with $e_1 = 24$, $e_2 = 6$ and $e_3 = -30$. Find a point $P \in E(\mathbb{Q})$ such that $\kappa(P) = (-1, -2, 2)$ and conclude that the rank of $E(\mathbb{Q})$ is at least 1.