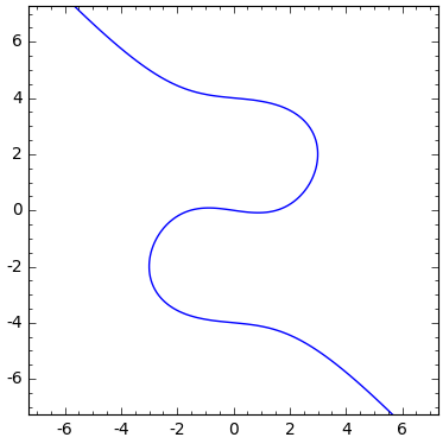
Coursework 1 on chapters 1, 2 and 3

1. Consider the equation

$$C : 3\,x^3 + xy^2 + 3\,y^3 - 7\,x - 48\,y = 0$$

illustrated in this picture:



We spot the solutions $P = (0, 0)$ and $Q = (3, 2)$.

a) Use the chord through P and Q to find a new solution.

b) The line $x = 3$ happens to be the tangent at Q to C. Use it to find a new solution R.

c) The line $y = -\frac{7}{48}\,x$ is the tangent at P to C. Do we get a new solution here?

2. Let $P = (1 : 1 : 1)$ and $Q = (3 : -4 : 1)$ in $\mathbb{P}^2(\mathbb{Q})$.

a) Find the projective line L passing through P and Q as a linear equation $aX + bY + cZ = 0$ by solving a system of linear equations as in the proof of Proposition 3.3.

b) Reformulate the affine piece of L in the form $y = m\,x + n$. Make a picture. Check that it passes through P and Q.

c) Write out a parametrisation $\ell(s : t)$ for this line L.

d) Use the very last equation in Section 3.5 to find a linear equation for L from the parametrisation.

3. Consider the line $L : X + 2Y + 3Z = 0$ and the line L' given by the parametrisation $\ell(s : t) = (s - 2t : -3s + 4t : 5s - 6t)$. Find the intersection $L \cap L'$.

4. Find the intersection of the lines given by the parametrisations $\ell_1(s : t) = (s : s + t : s + 7t)$ and $\ell_2(s : t) = (3t : 3s + t : s)$.