Self-points

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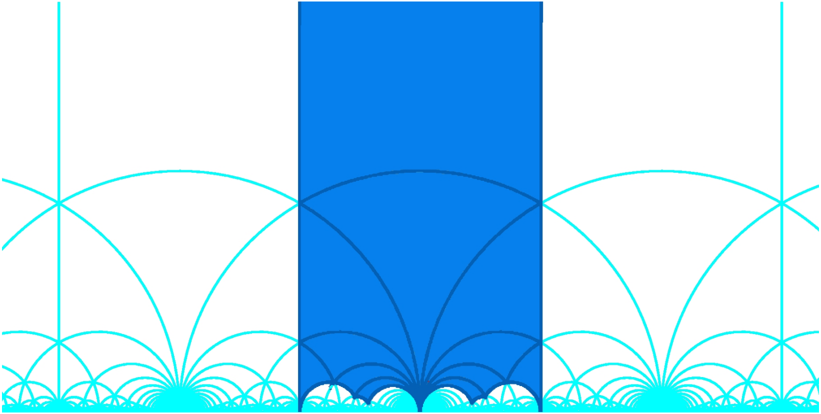
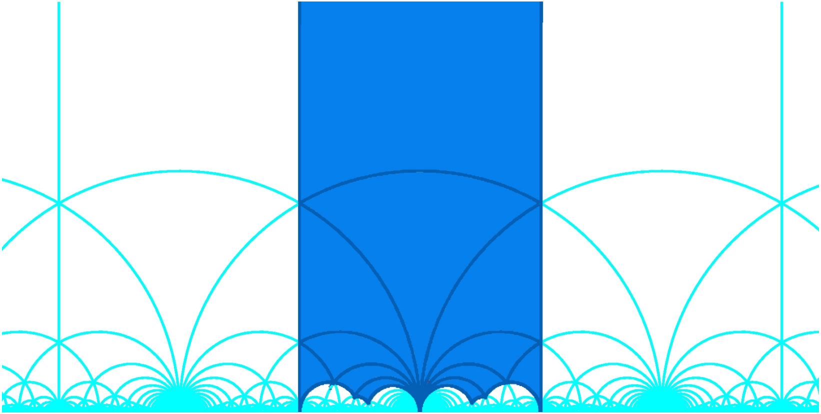
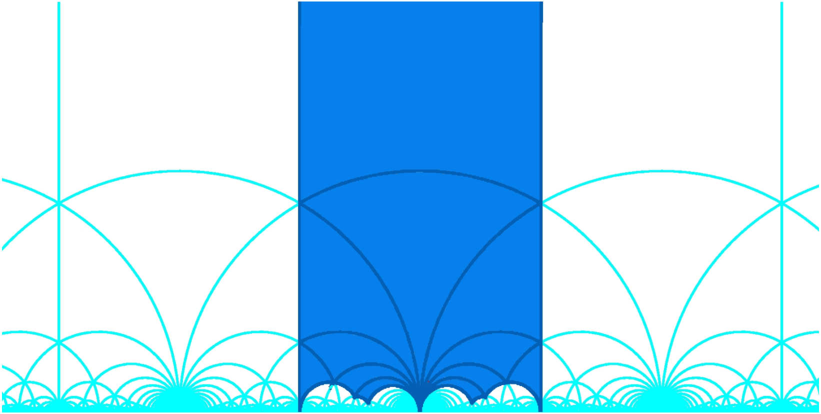


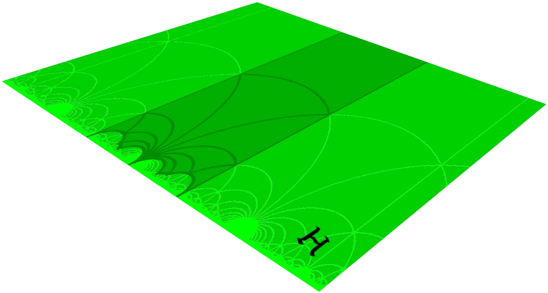


$$\tau \mapsto \frac{a\tau + b}{c\tau + d}$$



$$\Gamma_0(11) = \{ \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \mid ad - bc = 1,\ c \in 11\mathbb{Z},\ a, b, d \in \mathbb{Z} \}$$

$$\tau \mapsto \frac{a\tau + b}{c\tau + d}$$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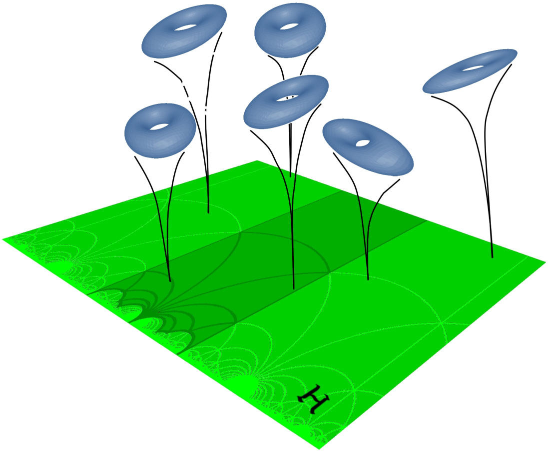
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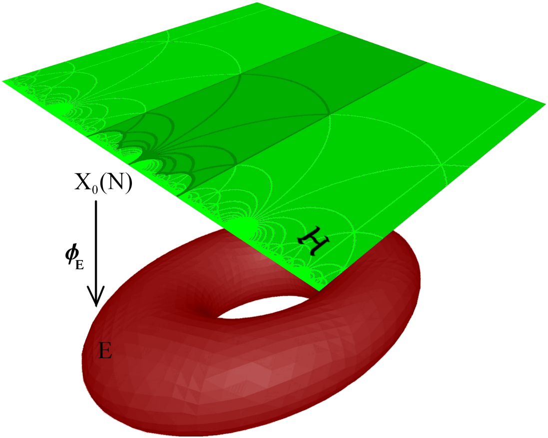
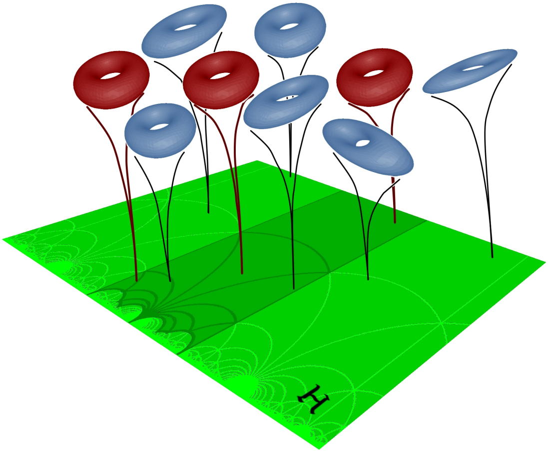


$X_0(N)$
$\mathcal{H}$
ϕ_E
E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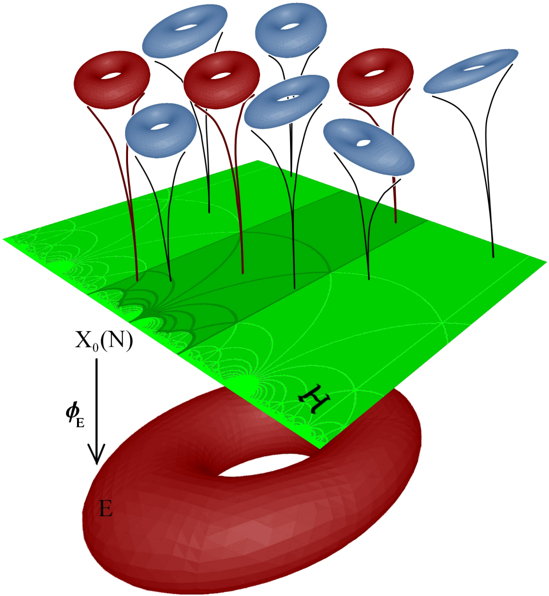
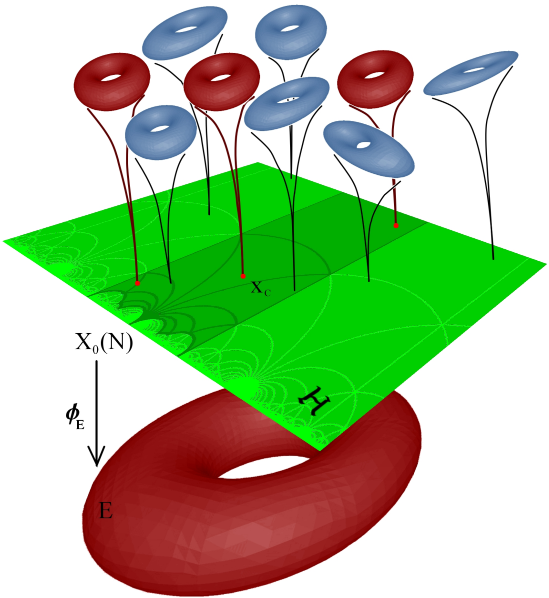
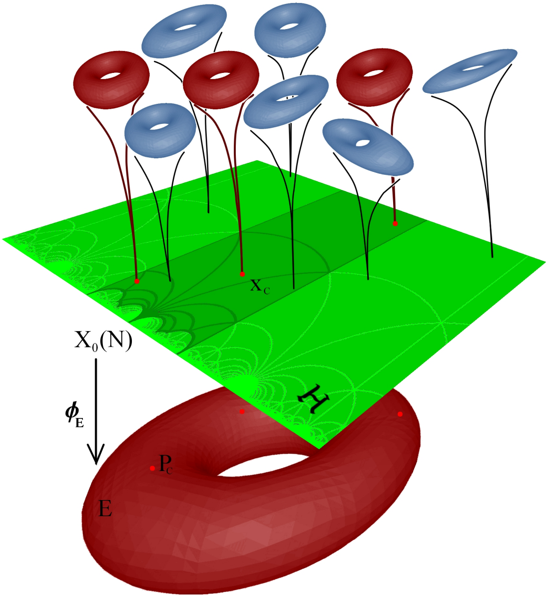


X0(N)
φE
H
E


X_C
$X_0(N)$
ϕ_E
$\mathcal{H}$
E



x_c
$X_0(N)$
ϕ_E
$\mathcal{H}$
P_c
E

The curve E is given by

$$y^2 + y = x^3 - x^2 - 10 \cdot x - 20$$

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Over $\mathbb{Q}$ there are only five solutions

$$\{(5,5),\ (16,-61),\ (16,60),\ (5,-6), (\infty,\infty)\}.$$

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$$y^2 + y = x^3 - x^2 - 10 \cdot x - 20$$

The points P_C are algebraic solutions over the field $\mathbb{Q}(\theta)$ generated by a solution θ of

$$\theta^{12} - 4\,\theta^{11} + 55\,\theta^9 - 165\,\theta^8 + 264\,\theta^7$$
$$-341\,\theta^6 + 330\,\theta^5 - 165\,\theta^4 - 55\,\theta^3$$
$$+99\,\theta^2 - 41\,\theta - 111 = 0\,.$$

$$
\begin{aligned}
x(P_c) = 2^{-1} \cdot 5^{-3} \cdot 11^{-10} \cdot 19^{-1} \cdot \\
\cdot (1337928020779520 89\, \theta^{11} \\
-3128489450052833 68\, \theta^{10} \\
-502878903201648831\, \theta^{9} \\
+6475439902255323868\, \theta^{8} \\
-11358894741986615604\, \theta^{7} \\
+17292758289068725628\, \theta^{6} \\
-18719462641364369973\, \theta^{5} \\
+16016249446153991254\, \theta^{4} \\
+982764516960529358\, \theta^{3} \\
-2408156353187544234\, \theta^{2} \\
+834424932645994748 3\, \theta \\
+791455726156281126 2) \,.
\end{aligned}
$$