

# Numerical modular symbols: Examples

Christian Wuthrich

May 3, 2017

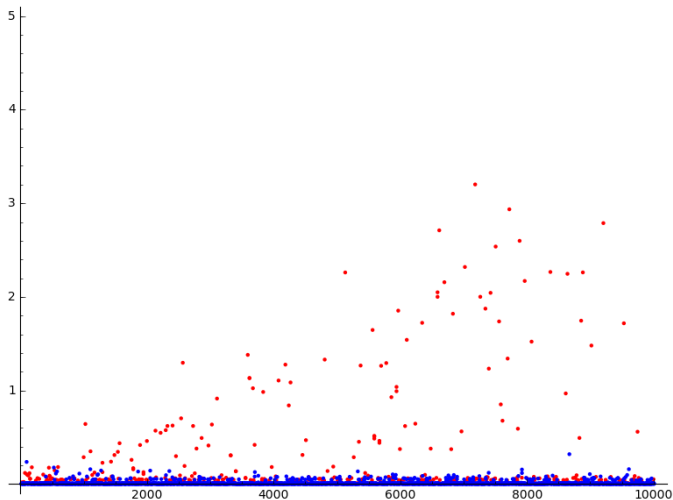
## Sage

```
E = EllipticCurve("234446a1")
m = E.modular_symbol(implementation="num")
m.all_values_for_one_denominator(25)
```

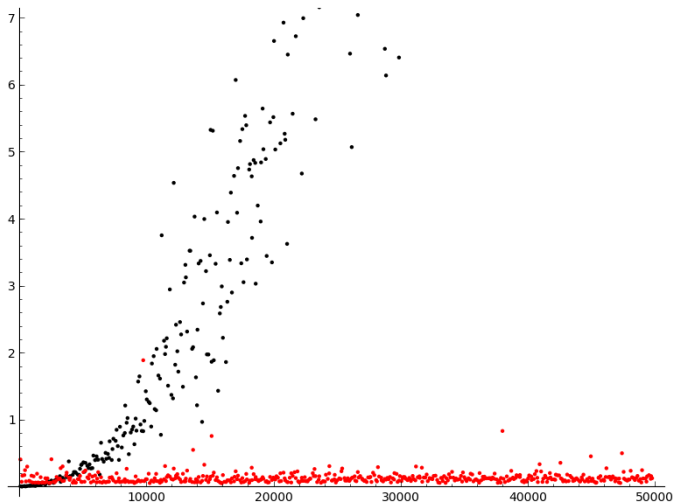
In less than 1 second, one gets all  $[\frac{a}{25}]$  and concludes that

$$L_5(E) = (2 + O(5^1)) \cdot T^4 + O(5^1) \cdot T^3 + O(5^1) \cdot T^2 + O(5^1) \cdot T$$

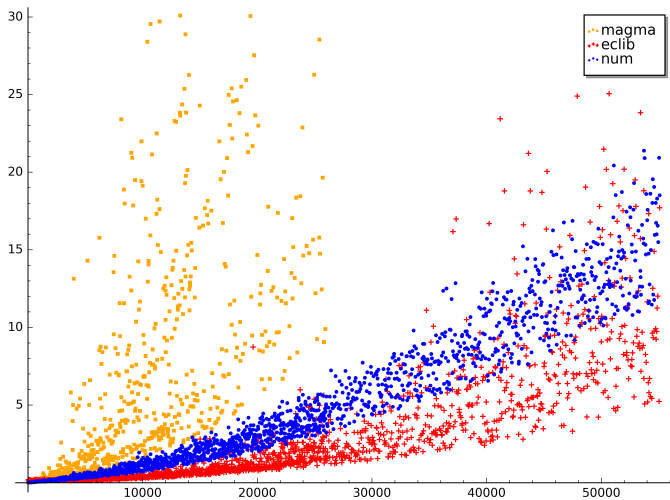
which shows that  $\text{rk } E(\mathbb{Q}) \leq 4$  and  $\text{III}(E/\mathbb{Q})[5] = 0$ .



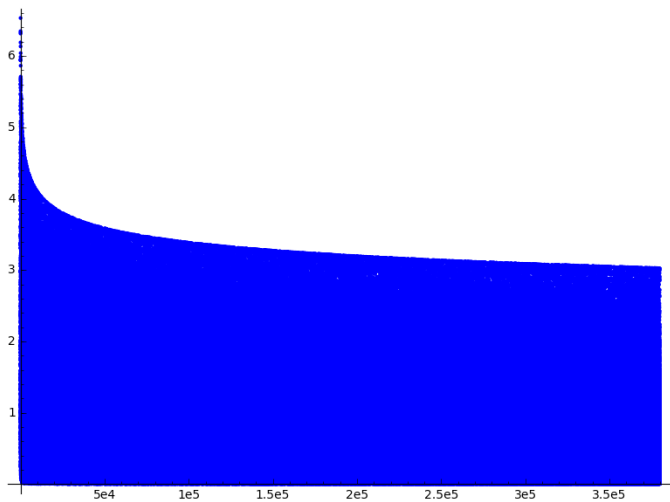
Compute  $[\frac{1}{9}]$  for random curves of small conductor.  
Red = Non-semistable curves.



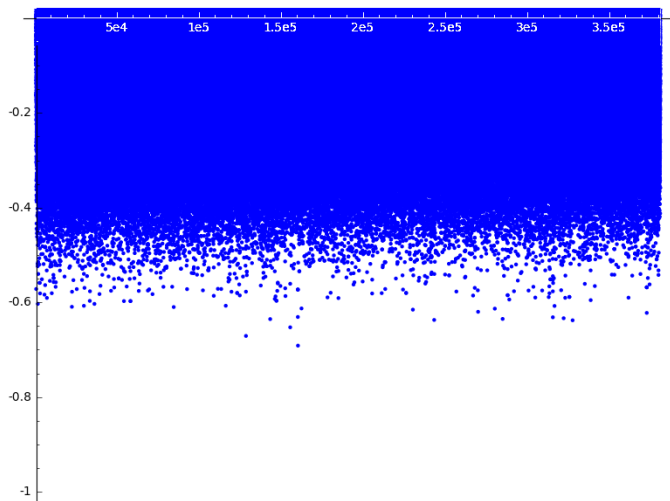
Compute one symbol (compared with Cremona's eclib)



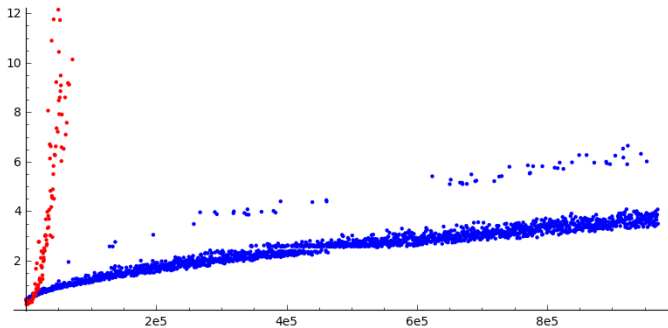
Compute all Manin symbol



$\Omega^+$  for all curves in the table



Values of  $k$  such that  $\Omega = N^k$ .



Compute  $L_5(T)$