SOME RECENT THEOREMS IN COMBINATORIAL ANABELIAN GEOMETRY

Non-trivial applications of the theory of "Topics Surrounding the Combinatorial Anabelian Geometry of Hyperbolic Curves II: Tripods and Combinatorial Cuspidalization", by Yu. Hoshi and Sh. Mochizuki, `http://www.kurims.kyoto-u.ac.jp/preprint/file/RIMS1870.pdf`

Theorem (Hoshi–Mochizuki–Minamide). *Let X be the projective line minus three points over an algebraic closed field of characteristic 0. Let X_2 be the second configuration space of X, i.e. the product X with itself minus the diagonal. Let Π_2 be the étale fundamental group $\pi_1(X_2)$ of X_2.*

Then its outomorphism group $\mathrm{Out}(\Pi_2)$ decomposes as the product of the profinite Grothendieck–Teichmüller group GT and the symmetric group S_5:

$$\mathrm{Out}(\Pi_2) = \mathrm{GT} \times S_5.$$

Theorem (Minamide–Nakamura). *Let $n \geqslant 4$.*

Then the outomorphism group of the profinite completion of the Artin braid group with n strings is isomorphic to the product of GT and the kernel of the natural projection $\hat{\mathbb{Z}}^{\times} \to (\mathbb{Z}/n(n-1)\mathbb{Z})^{\times}$.

Theorem (Tsujimura). *Let GT_p be the p-adic version of the Grothendieck–Teichmüller group defined using the tempered fundamental group.*

Then there exists a surjection $\mathrm{GT}_p \to G_{\mathbb{Q}_p}$ whose restriction to $G_{\mathbb{Q}_p}$ is the identity automorphism.

Theorem (Hoshi–Mochizuki–Tsujimura). *Let $\mathbb{Q}^{\mathrm{ab}}$ be the maximal abelian extension of $\mathbb{Q}$.*

Then the normaliser of $G_{\mathbb{Q}^{\mathrm{ab}}}$ in GT is the absolute Galois group $G_{\mathbb{Q}}$.

Theorem (Hoshi–Mochizuki–Tsujimura). *Let K be a finite extension of $\mathbb{Q}^{\mathrm{ab}}$. Let X,Y be hyperbolic curves over K of genus 0. Let X_2 be the second configuration space of X, i.e. the product X with itself minus the diagonal. Let Y_2 be the second configuration space of X, i.e. the product Y with itself minus the diagonal.*

Then the set of isomorphisms between X_2 and Y_2 is in bijection with the set of equivalence classes of isomorphisms between $\pi_1(X_2)$ and $\pi_1(Y_2)$ modulo the inner action of $\pi_1(Y_2)$.