

Author's comments. There are one or two comments I neglected to make in the course of the lecture or matters that were inadequately explained.

(i) The functions to be used in the proof proposed for functoriality are, at least initially, sums of logarithmic derivatives of automorphic L -functions,

$$-\sum_{\pi^{\text{st}}} m(\pi^{\text{st}}) \frac{L'(s, \pi, \rho)}{L(s, \pi, \rho)} = \sum_{n=1}^{\infty} \frac{\text{tr}_{\text{st}}(f_n)}{n^s},$$

so that n actually runs over powers p^k , $k > 0 \in \mathbb{Z}$, of primes. If the base field is not \mathbb{Q} , it runs over powers of prime ideals \mathfrak{p} . Indeed finitely many p , p_1, \dots, p_k , or finitely many \mathfrak{p} , $\mathfrak{p}_1, \dots, \mathfrak{p}_k$ may be excluded. The function f_{p^k} is the convolution of a spherical function at p and an arbitrary smooth, compactly supported function in $\prod_{k=1}^s G(F_{p^k})$, inserted so that the final result of the limiting process is applicable to individual classes of (cuspidal) automorphic representations. This second function is independent of n . The stable multiplicity $m(\pi^{\text{st}})$ is the multiplicity of the L -packet. The simplest example of this multiplicity is provided by $SL(2)$ and is discussed in the paper of Labesse-Langlands. The stable trace is that given by the stable trace formula, something whose existence I take for granted here!

(ii) Although I did not insist on it in the lecture, the notion of stable transfer, introduced with the example of $SL(2)$ in the paper *Singularités et transfert*, will be, I am convinced, an important feature of the trace formula.

(iii) I observe that no notion of reciprocity is suggested for the geometric theory, neither over a finite field nor over \mathbb{C} . One might ask whether this might exist and might involve one-parameter families of motives. I have never done so!

(iv) I believe I was sufficiently clear in the videos. I repeat here none the less that although the suggestions implicit in (ii) and (iii) demand a broad understanding of a number of different fields and, for some, a conceptual veering, as well as time and effort, they are not excessively daunting, whereas (i) and (iv) will demand more courage and talent than many of us have. I would hesitate to make, at this point, any very precise assertion, but as I reflect on the lecture the possibility of a more intimate relation than first envisaged between the Yang-Mills connections and the connections constructed from the Hecke eigenfunctions begins to suggest itself. I am still very uneasy with the former. It may be that the relation not only exists but also is well-known in some circles. At the moment, however, I have no reason to think so.

(v) Although Hecke operators appear nowhere in it, I have found the paper of Atiyah-Bott otherwise an excellent, as well as a rich, source of information about the geometry of Bun_G and the connections on the two spaces Bun_G and Bun_{L_G} . It is not, however, easy to read, and for anyone who does not possess a good deal of geometric and topological facility even more difficult. Moreover, the authors have been intimidated by the algebraic geometers' emphasis on stable bundles, so that some aspects of the geometry that are important in the context of the videos are not discussed. It is, nevertheless, instructive to be forced to reflect on these matters on one's own.

(vi) The purpose of my final comments was perhaps not clear. I am repeating a suggestion made elsewhere, also by me, that what is now, unfortunately and inappropriately, often

referred to as the Satake parameter be called the Frobenius-Hecke parameter.

(vii) I draw attention here to an absence in the video, that is also an absence in the “program.” Although I began the study of automorphic forms on listening to the lectures of Steven Gaal on a paper of Selberg’s, I was soon led to the papers of Siegel and Harish-Chandra. Among other things, Siegel’s papers were often concerned, in one way and another, with theta-series. His techniques were formalized by Weil as the symplectic group, a topic that was very popular for a while in the sixties and seventies of the last century, partly because it allowed the proof of functoriality (under the unfortunate label “lifting”) in a number of special cases. With the appearance of the notion of functoriality and the possibility of using the trace formula to prove it, the symplectic group has faded into the background, in part, at least in my view, rightly so. It and the associated automorphic forms of half-integral weight are nevertheless there and are not accounted for by the theory of automorphic forms adumbrated in the video. So something appears to be, or may be, lacking.