

RIMS/Symmetries and Correspondences Workshop: Combinatorial Anabelian Geometry and Related Topics

Organizers: Yuichiro Hoshi (RIMS, Kyoto University)
 Shinichi Mochizuki (RIMS, Kyoto University)
 Ivan Fesenko (The University of Nottingham)
 Arata Minamide (RIMS, Kyoto University)
 Shota Tsujimura (RIMS, Kyoto University)
 Benjamin Collas (RIMS, Kyoto University/The University of Lille)

Dates: July 5 – July 9, 2021
 Place: RIMS, online

	July 5 (Mon.)	July 6 (Tues.)	July 7 (Wed.)	July 8 (Thu.)	July 9 (Fri.)
16:30 – 17:30	[SemiAnab] (Yamashita)	[CmbCsp] (Minamide)	[CbTpII] (Hoshi)	BGT (Tsujimura)	[CbTpIII] (Hoshi)
17:50 – 18:50	[CmbGC] (Yamashita)	[NodNon] (Minamide)	[CbTpII] (Hoshi)	BGT (Tsujimura)	Recon. cusps (Iijima)
19:10 – 20:10	Overview (Mochizuki)	[CbTpI]/Hurwitz (Tsujimura)	[HMM] (Minamide)	Geo. pro- p (Higashiyama)	Pos. char. (Yang)
20:30 – 21:30	Overview (Mochizuki)	Lie alg. (Sawada)	Q & A (Mochizuki)	Geo. pro- p (Higashiyama)	Res. of nonsing. (Lepage)

Program

July 5 (Monday)

16:30 – 17:30 Go Yamashita (RIMS, Kyoto University)
Semi-graphs of Anabelioids

17:50 – 18:50 Go Yamashita (RIMS, Kyoto University)
A Combinatorial Version of Grothendieck Conjecture

19:10 – 20:10 Shinichi Mochizuki (RIMS, Kyoto University)
Overview of Combinatorial Anabelian Geometry I

20:30 – 21:30 Shinichi Mochizuki (RIMS, Kyoto University)
Overview of Combinatorial Anabelian Geometry II

July 6 (Tuesday)

16:30 – 17:30 Arata Minamide (RIMS, Kyoto University)
The injectivity portion of combinatorial cuspidalization for FC-admissible outer automorphisms I

17:50 – 18:50 Arata Minamide (RIMS, Kyoto University)
The injectivity portion of combinatorial cuspidalization for FC-admissible outer automorphisms II

19:10 – 20:10 Shota Tsujimura (RIMS, Kyoto University)
Geometric version of the Grothendieck conjecture for universal curves over Hurwitz stacks

20:30 – 21:30 Koichiro Sawada (RIMS, Kyoto University)
Reconstruction of invariants of configuration spaces of hyperbolic curves from associated Lie algebras

July 7 (Wednesday)

16:30 – 17:30 Yuichiro Hoshi (RIMS, Kyoto University)
Combinatorial Anabelian Geometry in the Absence of Group-theoretic Cuspidality

17:50 – 18:50 Yuichiro Hoshi (RIMS, Kyoto University)
Synchronization of Tripods and Glueability of Combinatorial Cuspidalizations

19:10 – 20:10 Arata Minamide (RIMS, Kyoto University)
The Grothendieck-Teichmüller group as an open subgroup of the outer automorphism group of the étale fundamental group of a configuration space

20:30 – 21:30 Shinichi Mochizuki (RIMS, Kyoto University)
Q & A

July 8 (Thursday)

16:30 – 17:30 Shota Tsujimura (RIMS, Kyoto University)
Combinatorial Belyi cuspidalization (I)

17:50 – 18:50 Shota Tsujimura (RIMS, Kyoto University)
Combinatorial Belyi cuspidalization (II)

19:10 – 20:10 Kazumi Higashiyama (RIMS, Kyoto University)
The mono-anabelian geometry of geometrically pro- p arithmetic fundamental groups
of associated low-dimensional configuration spaces II

20:30 – 21:30 Kazumi Higashiyama (RIMS, Kyoto University)
The mono-anabelian geometry of geometrically pro- p arithmetic fundamental groups
of associated low-dimensional configuration spaces III

July 9 (Friday)

16:30 – 17:30 Yuichiro Hoshi (RIMS, Kyoto University)
Applications to the Theory of Tempered Fundamental Groups

17:50 – 18:50 Yu Iijima (Hiroshima University)
On the centralizer of the image of the universal outer monodromy representation of
the moduli stack of pointed hyperbolic curves

19:10 – 20:10 Yu Yang (RIMS, Kyoto University)
Reconstructions of geometric data of pointed stable curves in positive characteristic

20:30 – 21:30 Emmanuel Lepage (IMJ, Sorbonne University)
Resolution of non-singularities

Abstracts

Speaker: Kazumi Higashiyama

Title: The mono-anabelian geometry of geometrically pro- p arithmetic fundamental groups of associated low-dimensional configuration spaces II, III

Abstract: Let p be a prime number. In this talk, we study geometrically pro- p arithmetic fundamental groups of low-dimensional configuration spaces associated to a given hyperbolic curve over an arithmetic field such as a number field or a p -adic local field. Our main results concern the group-theoretic reconstruction of the function field of certain tripods (i.e., copies of the projective line minus three points) that lie inside such a configuration space from the associated geometrically pro- p arithmetic fundamental group, equipped with the auxiliary data constituted by the collection of decomposition groups determined by the closed points of the associated compactified configuration space. I will give a lecture of part I in 2 July, 2021.

Speaker: Yuichiro Hoshi

Title: Combinatorial Anabelian Geometry in the Absence of Group-theoretic Cuspidality

Abstract: The main topic of this talk is a combinatorial anabelian result which allows one to eliminate the hypothesis that cuspidal inertia subgroups are preserved by the isomorphisms under consideration. This result leads us to partial generalizations of combinatorial cuspidalization results to the case of outer automorphisms of the fundamental groups of configuration spaces that do not necessarily preserve the cuspidal inertia subgroups of the various one-dimensional subquotients of such a fundamental group.

Speaker: Yuichiro Hoshi

Title: Synchronization of Tripods and Glueability of Combinatorial Cuspidalizations

Abstract: The main topic of this talk is the phenomenon of tripod synchronization, i.e., roughly speaking, the phenomenon that an outer automorphism of the fundamental group of a log configuration space associated to a stable log curve typically induces the same outer automorphism on the various subquotients of such a fundamental group determined by tripods. We also discuss the theory of glueability of combinatorial cuspidalizations, i.e., roughly speaking, the theory which shows that the condition for an outer automorphism of the fundamental group of a stable log curve to lift to an outer automorphism of the fundamental group of the corresponding log configuration space is compatible, in a suitable sense, with localization on the dual graph of the stable log curve.

Speaker: Yuichiro Hoshi

Title: Applications to the Theory of Tempered Fundamental Groups

Abstract: The main topic of this talk is the issue of applications of the theory developed in combinatorial anabelian geometry to the theory of tempered fundamental groups in the style of André. The characterization of the local Galois groups in the global Galois image associated to a hyperbolic curve that is given in earlier work of André is only proven for a quite limited class of hyperbolic curves, i.e., a class that is “far from generic”. By combining the notion of M-admissibility with some combinatorial anabelian results and

techniques, together with the theory of cyclotomic synchronization, we obtain a generalization of André's characterization of the local Galois groups to the case of arbitrary hyperbolic curves.

Speaker: Yu Iijima

Title: On the centralizer of the image of the universal outer monodromy representation of the moduli stack of pointed hyperbolic curves

Abstract: As an application of the combinatorial anabelian geometry, under a condition concerning cuspidal inertia subgroups, Y. Hoshi and S. Mochizuki calculated centralizers of images of geometric outer monodromy representations of certain families of hyperbolic curves. In this talk, I discuss this condition.

Speaker: Emmanuel Lepage

Title: Resolution of non-singularities

Abstract: A hyperbolic curve over \mathbb{C}_p satisfies resolution of non-singularities if quotients by Galois group of stable models of finite étale Galois covers are cofinal among semistable models. This technical property has several anabelian applications. In this talk we will explain why Mumford curves satisfy resolution of non-singularities.

Speaker: Arata Minamide

Title: The injectivity portion of combinatorial cuspidalization for FC-admissible outer automorphisms I, II

Abstract: In this series of two talks, we will sketch the proof (due to Yuichiro Hoshi and Shinichi Mochizuki) of the injectivity portion of combinatorial cuspidalization for FC-admissible outer automorphisms. In this proof, combinatorial versions of the Grothendieck Conjecture play essential roles. This injectivity allows one to generalize a certain well-known injectivity theorem of Belyi to the case of arbitrary hyperbolic curves.

Speaker: Arata Minamide

Title: The Grothendieck-Teichmüller group as an open subgroup of the outer automorphism group of the étale fundamental group of a configuration space

Abstract: Let $n \geq 2$ be an integer and k an algebraically closed field of characteristic zero. Write Π_n for the étale fundamental group of the n -th configuration space of the projective line minus $\{0, 1, \infty\}$ over k , GT for the (profinite) Grothendieck-Teichmüller group, and \mathfrak{S}_{n+3} for the symmetric group on $n + 3$ letters. In this talk, we will discuss a result to the effect that the natural outer actions of GT and \mathfrak{S}_{n+3} on Π_n determine an isomorphism $\text{GT} \times \mathfrak{S}_{n+3} \xrightarrow{\sim} \text{Out}(\Pi_n)$. This leads to a simple purely group-theoretic characterization of GT. This is joint work with Yuichiro Hoshi and Shinichi Mochizuki.

Speaker: Shinichi Mochizuki

Title: Overview of Combinatorial Anabelian Geometry I & II

Abstract: Combinatorial anabelian geometry may be described as the study of the combinatorial and group-theoretic properties of semi-graphs of anabelioids that arise from

stable log curves and their associated log configuration spaces. In these talks, we survey many (but not all!) important developments in combinatorial anabelian geometry during the last 15 years, focusing on the exposition of the main ideas from an intuitive, “pictorial” (as opposed to technically precise) point of view. The exposition will center around the following topics:

- (T1) motivation from differential topology and arithmetic ([CbCsp]),
- (T2) combinatorial anabelian results ([CbGC], [NodNon], [CbTbII]),
- (T3) combinatorial cuspidalization and “FC = F” results ([CbCsp], [NodNon], [CbTpI], [CbTpII]),
- (T4) arithmetic applications of combinatorial cuspidalization over number fields and mixed-characteristic local fields ([NodNon], [CbTpIII]),
- (T5) cyclotome synchronization, profinite Dehn twists, and geometry monodromy anabelian results ([CbTpI]),
- (T6) tripod synchronization and the tripod homomorphism ([CbTpII]),
- (T7) structure theory and arithmetic subgroups of the Grothendieck-Teichmüller group ([HMM], [HMT]).

Speaker: Koichiro Sawada

Title: Reconstruction of invariants of configuration spaces of hyperbolic curves from associated Lie algebras

Abstract: Hoshi, Minamide, and Mochizuki gave certain explicit group-theoretic algorithm for reconstructing some objects from the étale fundamental group of the configuration space of a hyperbolic curve over an algebraically closed field of characteristic zero. In this talk, we consider “Lie algebra analogues” of these reconstruction algorithms.

Speaker: Shota Tsujimura

Title: Geometric version of the Grothendieck conjecture for universal curves over Hurwitz stacks

Abstract: Y. Hoshi and S. Mochizuki introduced and developed the theory of profinite Dehn twists in their study of combinatorial anabelian geometry and applied it to prove the geometric version of the Grothendieck conjecture for universal curves over moduli stacks of hyperbolic curves. In this talk, after reviewing the theory of profinite Dehn twists briefly, we discuss the Hurwitz stack version of this application.

Speaker: Shota Tsujimura

Title: Combinatorial Belyi cuspidalization (I)

Abstract: S. Mochizuki developed the theory of Belyi cuspidalization and applied it to reconstruct the decomposition groups associated to the closed points of hyperbolic curves of strictly Belyi type over p -adic local fields. In this talk, we first discuss a certain combinatorial version of the theory of Belyi cuspidalization, which is based on the theory of tripod synchronization [developed by Y. Hoshi and S. Mochizuki]. After that, we apply the theory of combinatorial Belyi cuspidalization, together with E. Lepage’s result on resolution of nonsingularities, to construct a splitting of the natural injection from the absolute Galois group of the field of p -adic numbers to the p -adic version of the Grothendieck-Teichmüller group [introduced by Y. Andre].

Speaker: Shota Tsujimura

Title: Combinatorial Belyi cuspidalization (II)

Abstract: Subsequent to the previous talk, we discuss another application of the theory of combinatorial Belyi cuspidalization. In this talk, we give a purely combinatorial/group-theoretic construction of the conjugacy class of subgroups of the Grothendieck-Teichmüller group determined by the absolute Galois group of the field of rational numbers. Moreover, we apply this construction to obtain a semi-absolute version of the Grothendieck conjecture for higher dimensional configuration spaces associated to hyperbolic curves of genus zero over TKND-AVKF-fields [for instance, the maximal cyclotomic extensions of number fields]. This is joint work with Y. Hoshi and S. Mochizuki.

Speaker: Go Yamashita

Title: Semi-graphs of Anabelioids

Abstract: I explain Mochizuki's paper "Semi-graphs of Anabelioids".

Speaker: Go Yamashita

Title: A Combinatorial Version of Grothendieck Conjecture

Abstract: I explain Mochizuki's paper "A Combinatorial Version of Grothendieck Conjecture".

Speaker: Yu Yang

Title: Reconstructions of geometric data of pointed stable curves in positive characteristic

Abstract: Let (X, D_X) be a pointed stable curve of type (g, n) over an algebraically closed field k of characteristic $p > 0$. In the talk, I will explain that many geometric data associated to (X, D_X) can be reconstructed group-theoretically from its geometric fundamental group. The results was proved by A. Tamagawa 20 years ago when X is smooth over k , and by the speaker when X is an arbitrary pointed stable curve.