ABOUT CERTAIN ASPECTS OF THE STUDY AND DISSEMINATION OF
SHINICHI MOCHIZUKI’S IUT THEORY

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“Phil: Do you ever have deja vu, Mrs. Lancaster?
Mrs. Lancaster: I don’t think so, but I could check with the kitchen.”
(Groundhog Day)

This text communicates in a compact form some of factual information related to the study of Sh. Mochizuki’s
IUT theory1 and its dissemination, as well as various aspects of the situation around IUT. These facts might be
not terribly well known to some mathematicians, especially outside arithmetic geometry.

IUT was made public in August 2012. More general issues are discussed in two other papers2,3. Without
repeating the content of those papers, this text deals with some concrete issues and also includes some
recommendations.

In summary of what is discussed below, these are the facts. The main prerequisite for IUT is the huge area of
(arithmetic) anabelian geometry developed in 1990–2016 in Japan. In 2012 there were few experts in anabelian
geometry outside Japan. IUT is not an increase of mathematical knowledge in an area in which there are many
specialists able to study it. It is a very novel development with many new concepts. To become an expert in
IUT, one has to invest an adequate large amount of time in a dedicated serious focused study of the theory
starting with its prerequisites.

To help mathematicians to study IUT, a vast amount of time and effort has been dedicated to the dissemina-
tion of IUT, via various workshops, including large international, via seminars, lectures and study groups. The
number of researchers who have mastered IUT is steadily growing and is a two-digit one. These researchers
have sent more than 1000 questions and remarks to the author, all addressed. No valid math evidence of any
serious fault in IUT has been found. Minor oversights have been found and corrected and to this day there
remains no mathematically substantive reason whatsoever to doubt the validity of IUT. Seven surveys of IUT
have already been written, each individually presenting the theory in a different way. The number of mathe-
maticians able to write expert reports on the IUT papers exceeds the number of such reports on previous major
breakthrough papers at the time of their publication. 2020 will be a special RIMS year with 4 international
workshops on anabelian geometry, combinatorial anabelian geometry and IUT.

Some mathematicians have tried to study IUT on the own, but have not been able to proceed far. In particular,
none of number theorists who made their own breakthrough decades ago have apparently managed to advance
in their study of IUT. This fact can be nicely contrasted with the fact that there are several young researchers

1 The IUT papers are available from section Inter-universal Teichmüller Theory of its author page http://www.kurims.kyoto-u.
ac.jp/~motizuki/papers-english.html. See those pages for various information on seminars and workshops on IUT. See also
this page https://www.maths.nottingham.ac.uk/plp/pmzibf/guidestoiut.html.
2 I. Fesenko, Arithmetic deformation theory via arithmetic fundamental groups and nonarchimedean theta functions, notes on the
pmzibf/notesonIUT.pdf
ac.uk/plp/pmzibf/rapm.pdf
who in the course of several years diligently studied IUT, asked interesting questions, and contributed to new original developments.

There were few people active in applying efforts to produce ignorant remarks about IUT without having any expertise in the subject area and without being able to indicate any valid math evidence of faults in the theory. Some of them were active in spreading fake information on the internet, which might have affected some mathematicians in other areas.

1. On mathematical environment around IUT, briefly. Class field theory, the heart of algebraic number theory, has several important generalisations. They include the Langlands correspondences, anabelian geometry and higher class field theory. By various reasons the first generalisation has attracted many times more researchers than the second and the third, but all of these generalisations of class field theory are fundamentally important. The main conjectures over number fields in the first generalisation of class field theory remain open, despite the well known achievements in some special cases and fundamental advances in the functional and geometric cases. Most of the central problems in the second and third generalisations of class field theory have been settled\(^4\) even though the numbers of researchers working there is much smaller than in the Langlands correspondences.

The main prerequisite for IUT theory of Sh. Mochizuki is his arithmetic anabelian geometry. In other words, it is anabelian geometry (as started by Neukirch–Ikeda–Uchida–Iwasawa and from a different motivation for hyperbolic curves by Grothendieck) of small fields (such as number fields or their completions) in characteristic zero and of hyperbolic curves over small fields in characteristic 0. The main leading centre in arithmetic anabelian geometry is Japan.\(^5\) Below ‘anabelian geometry’ will mean ‘arithmetic anabelian geometry’. In the period of 1990–2016 a vast body of fundamentally important results in anabelian geometry were established. These developments were essentially left unnoticed outside the small group of experts.

The IUT theory\(^6\) uses some key theorems in anabelian geometry, as well as its later developments such as absolute anabelian geometry and mono-anabelian geometry. One starting observation for arithmetic deformation theory, i.e. IUT, is that unlike the usual algebraic geometry in which working with schemes corresponds to working with rings, working with certain anabelian objects corresponds to working with topological groups, thus one operation instead of two and more options to deform the objects. The total volume of relevant papers in anabelian geometry used in one or another extent in IUT is huge, even though it is possible not to read all of its 1500 pages.

2. The study of IUT. Links to various study materials about IUT are available from pages of the author of IUT\(^7\). IUT can be a difficult theory to study for experts from other areas, such as arithmetic of elliptic curves, Galois representations and diophantine geometry, since they cannot easily apply their previous expertise without learning first, as PhD students, new for them related areas starting with arithmetic anabelian geometry.

\(^4\) For more detail see https://www.maths.nottingham.ac.uk/plp/pmzibf/232.pdf.
\(^5\) In the 1990s, a series of results about anabelian properties of Galois groups of global and higher global fields, i.e. birational anabelian geometry, were obtained by F. Pop. Since the early 1990s, F. Bogomolov suggested and developed, later in collaboration with Yu. Tschinkel, his birational anabelian geometry for varieties of dimension > 1 over algebraically closed fields, this theory is quite different from arithmetic anabelian geometry in many respects.
\(^6\) Produced alone, compare: ‘They’ve all done things, often beautiful things, in a context that was already set out before them, which they had no inclination to disturb. Without being aware of it, they’ve remained prisoners of those invisible and despotic circles which delimit the universe of a certain milieu in a given era. To have broken these bounds they would have to rediscover in themselves that capability which was their birth-right, as it was mine: the capacity to be alone’, pp. 34–35 of the English transl. of A. Grothendieck’s ‘Récoltes et Semaines’, http://matematicas.unex.es/~navarro/res/lisker1.pdf.
\(^7\) http://www.kurims.kyoto-u.ac.jp/~motizuki/top-english.html
The total amount of time dedicated to the verification process of IUT by mathematicians already exceeds 30 researcher-years. This seems to be the largest time ever spent in the history of mathematics on the verification of mathematical work prior to its publication. Several international conferences were organised in 2014-2016. Numerous intensive seminars have been held in Japan (2012-2019), UK (2015-2018), China (2015-2016), they involved nationals of many countries. In addition to referees’ comments, active learners of IUT sent in 2012-2017 a 4-digit number of comments, questions, remarks, all had been carefully taken into account by the author. Several learners of IUT shared their understanding of it, by writing texts and surveys. There are already more surveys of IUT than of any previous fundamental work at the time of its publication.\(^8\) There will be 4 international workshops on anabelian geometry and IUT at RIMS in 2020.

The absence of experts in anabelian geometry worldwide, the lack of good digestion of Grothendieck’s heritage and the current situation with top research in number theory have substantially affected the ability of contemporaries to study IUT. Some people applied serious efforts to study IUT for some time, but stopped—indeed, the task is huge. Mathematicians who are used to understand standard/conventional things quickly and who write their papers fast but who are unable to work on the same topic for more than few weeks or months cannot progress in the study of IUT which is a long-term project. Several expected candidates to study the theory chose to be as unambitious as it can get, and to do essentially nothing for seven years. Some preferred to adopt the convenient stance of sceptical attitude not based on any expert knowledge of the subject area. Some chose to spread negative rumours, sometimes in order to justify their inability to study the theory or even to prepare for a workshop on IUT\(^9\). One of such false rumours is that there might be some kind of controversy about the status of IUT; to have a controversy there should be genuine experts on both sides of the argument, but this is not the case.

**Recommendations to mathematicians who are interested to study IUT:** Every serious researcher, ready for a longer-term study, is welcome to join the study and developments of IUT. Pathways to study IUT are available from many sources including www-links in footnotes of this text. If you find a piece of IUT looking to you as an error, and you cannot resolve it, document your evidence and contact the author or experts to discuss. Some may mention it is too difficult to study IUT, but we have numerous examples of young researchers going through it. Some may be waiting for a baby food, ready-to-digest version of the theory but it will take time for it to be produced.\(^{10}\) If you meet a mathematician spreading rumours about something wrong in IUT, before starting to believe their information, check if it is the first-hand knowledge or not, check that person level of expertise in anabelian geometry and ask for a concrete documented math evidence of any fault in IUT.

3. The reaction to IUT.

3.1. On some of reaction to IUT by mathematicians. Who can doubt that any professional consensus about any mathematical theory can only come from experts in its subject area. It is crucial to appreciate that whatever are one’s previous great results in mathematics, the time issue and other aspects indicate that the ability to learn a new groundbreaking theory requires different people, and the history teaches us that younger researchers are often do much better in studying fundamentally new developments. Thus, instead of waiting for opinions of so called ‘influential mathematicians’, one can listen to new rising stars who have mastered the theory.

Mochizuki’s work includes fundamental pioneering contributions in numerous directions: Hodge–Arakelov theory, anabelian geometry, mono-anabelian geometry, combinatorial anabelian geometry, Grothendieck–Teichmüller group, p-adic Teichmüller theory, inter-universal Teichmüller theory. Except for the last direction, none

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\(^8\) [http://www.maths.nottingham.ac.uk/plp/pmzibf/guidestoiut.html](http://www.maths.nottingham.ac.uk/plp/pmzibf/guidestoiut.html).

\(^9\) See e.g. the report about the Oxford IUT workshop [https://www.maths.nottingham.ac.uk/plp/pmzibf/files/iut-i-rep.html](https://www.maths.nottingham.ac.uk/plp/pmzibf/files/iut-i-rep.html)

\(^{10}\) the best presentation of Galois theory, due to E. Artin, appeared almost 100 years after the text of Galois
of his work has ever been criticised — because it was read and appreciated by experts. Some math countries
do not have any experts in arithmetic anabelian geometry and then local mathematicians could be misled by
irresponsible rumours spread originating, by various reasons, from some non-experts. Talking negatively about
IUT always encounters the glaring problem: the sheer inability to indicate any concrete valid error or ask at
least some good questions about IUT.

A. Beilinson, 2018 Wolf Prize winner, wrote 'I believe that in mathematics, as everywhere else, you can say
that something is correct or not only if you have understood this yourself. Since we do not have time to do
everything, in mathematics I tend to believe that something is correct if I can understand some pieces of the
proof or theory. If I do not understand anything, I try to refrain from making judgement.'

This attitude is shared by most mathematicians. It is strange that it is not shared by some who are keen to
make their ignorant opinions public. See also 3.3 and 3.4.

3.2. Articles about IUT in mass media of some countries. IUT has attracted a high level of interest from
mass media. There are some reasonably good written articles about IUT and its author. At the same time, there
are irresponsibly written articles presenting very inaccurate pictures.

Most experts on IUT decline to answer journalists questions, so then journalists contact mathematicians or
even laypersons with zero publication record in number theory. Some of the interviewed mathematicians are
good in their own areas, but that does not make them experts in areas they do not know. Experience in classical
Diophantine geometry, algebraic geometry, modularity, Galois representations or aspects of p-adic geometry
does not enable one with the expert intuition and knowledge of anabelian geometry and IUT.

One of standard ways for journalists to write their articles is to present opposite points of view but in the
case of IUT the journalists often fail to appreciate that they mix experts opinions (all of which are positive) with
negative or ignorant opinions of non-specialists. It is similar to as if an article about the value of a graduate
course is written by mixing opinions of its students with grade A and its students who never attended any lecture
and got grade F.

Recommendation to serious journalists: before interviewing a mathematician about IUT, first check the
expertise level by asking several simple questions such as their knowledge of and expertise in anabelian geom-
etry, including their publications record, the number of hours spent on the study of IUT and whether they asked
questions about IUT and related prerequisites to the author of IUT or experts in IUT.

3.3. An attempt to study IUT by two German mathematicians and ethical issues. In 2013–2017 not a
single concrete mathematical remark indicating a serious problem in IUT was produced. This did not prevent
some cheap irresponsible talk. Since 2014 P. Scholze kept talking publicly at various workshops about faults
in IUT. Eventually Scholze visited RIMS, together with J. Stix, in March 2018, just for 5 days. After the
meeting, Scholze and Stix came with their caricature version of IUT based on their oversimplification of IUT
in which they identify all isomorphic rings and ‘forget’ about the fundamental role of automorphism groups. In
particular, the two German mathematicians deny the use of anabelian geometry and infinitely many theatres in

11 personal communication, January 2018
12 The author of this text and some other people wrote to him several times asking to tell precisely what were the faults in IUT
he knew. He declined to participate in IUT workshops. Eventually, he sent just one most loosely stated question to Mochizuki in
May 2015, perhaps related to the so called Ind3 indeterminacy. Ind1-Ind3 are three fundamental indeterminacies in IUT one needs to
allow in order to have certain functoriality/multiradiality. The author of IUT responded to him with a long email which also offered to
conduct discussions via email to address any questions, but Scholze declined to communicate further. Part of this is stated on p.3 of
kyoto-u.ac.jp/~motizuki/IUTch-discussions-2018-03.html.
13 Mochizuki writes about this on the same p.3, ‘On the other hand, the March 2018 discussions centred around quite different
issues, such as (Ind1,2)’. Note: not Ind3 anymore.
IUT. Initially, Scholze and Stix intended to put their report about the meeting online. However, after reading Mochizuki’s report on their report, see especially its sect. 17-18 and these comments, they completely changed their mind in July 2018 and stopped to be interested to post their own report. They eventually agreed to let the author of IUT to include their report on his pages. The author of IUT formulated several questions to the German mathematicians in his report, which may have helped them to appreciate how erroneous was their take on IUT. The second version of their report did not address most of comments of Mochizuki on their first report. The second version of their report also included new incorrect statements such as a blunder in classical height theory and a fundamental misunderstanding of one of Faltings work. No mathematicians are known to support the superficial take of Scholze–Stix on IUT. Their short lived study of IUT stands in stark contrast with the deep study of it by the other mathematicians mentioned above, who asked/made many good questions, remarks and comments. The failure of those two German mathematicians should not stop serious researchers to study IUT.

The failure of Mrs. Lancaster to understand the question does not in any way imply anything negative about the question.

If one does not apply appropriate efforts to study the area of a fundamentally new theory, one does not become an expert in it, whatever one’s own different area of specialisation is and achievements in it. Of course, it is still possible to contribute useful questions, comments, remarks in relation to more conventional parts of the theory, e.g. those which came in 2012 from two analytic number theorists. To make a mistake in one’s mathematical study is rather normal, especially when one tries to understand a complex theory. However, to publicly talk about faults in another theory for several years and spread harmful rumours, without ever having any valid evidence of the faults, is irresponsible.

3.4. Some of online reaction to IUT. In the first approximation, the number of ignorant negative reactions to IUT was inversely proportional to the number of home academicians capable to study the theory. Something is fundamentally rotten here, and it has to be addressed properly. Why did these few mathematicians make public their opinions about a fundamental development in the subject area where they have empty research record, with no evidence of their serious study of it, and without providing any math evidence of errors in the theory? If they feel bad about their own inability to study the theory, it is still not the reason to talk rubbish. What are the real reasons of this unprofessional behaviour? If one is reckless, one should learn the error of making hasty judgments and come to appreciate the difference between the superficial and the essential. The less innocent reasons are pursuing goals having nothing to do with the theory they chose to say negative things about.

4. Developments. Some of them are mentioned above. The book by F. Kato about IUT provides more general information about various features of IUT to the wider audience. This book was in the list of top twenty bestselling books in all subject areas on amazon.co.jp and was awarded the Yaesu prize. There will be 4 international workshops during a special RIMS Project Research year on Expanding Horizons of Interuniversal Teichmüller Theory in 2020–2021. The new Center for Research in Next-Generation Geometry at RIMS supports these and other related developments.

\[\text{For a popular presentation to high school students of the importance to use infinitely many theatres in IUT, one can watch F. Kato’s talk https://www.youtube.com/watch?v=fNS7NO4DLAQ&vl=en and read his bestselling book https://twitter.com/FumiharuKato.}\]

\[\text{see footnote 12}\]

\[\text{http://www.kurims.kyoto-u.ac.jp/~motizuki/Cmt2018-05.pdf}\]

\[\text{In 1931 a group of scientists published a book ‘Hundert Autoren gegen Einstein’. This book is now viewed as ‘a reaction of an inadequately educated academic citizenship, which didn’t know what to do with relativity’ and as an ‘accumulation of naive errors’.}\]