This text aims to communicate in a compact form some of factual information related to the math study of Sh. Mochizuki’s IUT theory\(^1\) and its dissemination, as well as various aspects of the situation around IUT such as its perception by non-experts. Some of more general issues are discussed in two other papers\(^2\)\(^3\). Without repeating the content of those papers, this text deals with some concrete issues, rather related to mathematicians than to mathematics. It also includes some recommendations. The summary of what follows is this.

The main prerequisite for IUT is the vast area of (arithmetic) anabelian geometry developed in 1990–2016 in Japan. In 2012 there were few experts in anabelian geometry outside Japan. IUT is not an increase of mathematical knowledge in an area in which there are many specialists able to study it. It is a rare pioneering vast development with many new concepts and ideas, and with a great potential for future developments. To become an expert in IUT, one has to invest an adequate large amount of time in a dedicated serious focused study of the theory starting with its prerequisites. This cannot be done in the period of few weeks.

To help mathematicians to study IUT, a very large amount of time and effort has been dedicated to the dissemination of IUT, via various workshops, including large international, via seminars, lectures and study groups. No valid math evidence of any serious fault in IUT has been found. Minor oversights have been found and corrected. To this day there remains no mathematically substantive reason whatsoever to doubt the validity of IUT. The number of researchers who have mastered IUT, by investing a large amount of time and effort, is steadily growing and is a two-digit one. These researchers have sent a 4-digit number of questions and remarks to the author, all addressed. 11 text-surveys of IUT and a book on IUT by 9 mathematicians from 5 countries individually present the theory in different ways. There are more mathematicians able to produce expert reports on the IUT papers than the number of such reports on previous rare major breakthrough math results at the time of their publication. 2020–2021 is a special RIMS year with 4 international workshops on anabelian geometry, combinatorial anabelian geometry and IUT.

Some mathematicians have tried to study IUT on the own, but have not been able to proceed far. This is normal. In particular, none of number theorists who made their own genuine breakthrough decades ago have apparently managed to advance in their study of IUT. In interesting contrast, there are several young researchers

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\(^1\) The IUT papers made public in August 2012 are available from section Inter-universal Teichmüller Theory of its author page http://www.kurims.kyoto-u.ac.jp/~motizuki/papers-english.html. See those pages for various information on seminars and workshops on IUT. See also this page https://www.maths.nottingham.ac.uk/plp/pmzibf/guidestoIUT.html.


\(^3\) I. Fesenko, Remarks on aspects of modern pioneering mathematical research, available from https://www.maths.nottingham.ac.uk/plp/pmzibf/rapm.pdf
who in the course of several years of hard study of IUT, have asked interesting questions and contributed to new original developments.

In mathematics the only valid arguments in favour or against are always based on 100% math truth\(^4\), unlike most other areas of human knowledge. Unusually for mathematics, but perhaps reflecting the time we live in, there were few researchers, each lacking any expertise even in anabelian geometry, active in applying efforts to produce ignorant critical remarks about IUT, always devoid of any valid math evidence of faults in the theory. Some spread abject false online information about IUT that might have affected some mathematicians in other areas, unable to distinguish an expert in the relevant subject area from a non-expert.

1. **On mathematical environment around IUT, briefly.** Class field theory, the heart of algebraic number theory, has several important generalisations. They include the Langlands correspondences, anabelian geometry and higher class field theory. By various reasons the first generalisation has attracted many times more researchers than the second and the third, but all of these generalisations of class field theory are fundamentally important. Most of the central problems in the second and third generalisations of class field theory have been settled\(^5\). One can imagine another universe where higher class field theory and anabelian geometry attract many more researchers than in this universe and where general class field theory concepts are well understood and used for fundamental achievements in the Langlands program. In this universe the main conjectures over number fields in the Langlands program remain open, despite the well known achievements in some very special cases and fundamental advances in the functional and geometric cases.

The main prerequisite for IUT theory of Sh. Mochizuki is arithmetic anabelian geometry, including his famous proofs of the Grothendieck conjecture and his mono-anabelian geometry. Arithmetic anabelian geometry was started in works of Neukirch–Ikeda–Uchida–Iwasawa for small fields (such as number fields or their completions) in characteristic zero, and from a different motivation for hyperbolic curves over number fields it was proposed by Grothendieck. The main leading country in arithmetic anabelian geometry is Japan, and the first three contributors to anabelian geometry were H. Nakamura, A. Tamagawa and Sh. Mochizuki.\(^6\) Below ‘anabelian geometry’ will mean ‘arithmetic anabelian geometry’. In the period of 1990–2016 a vast body of fundamentally important results in anabelian geometry were established. These developments were essentially left unnoticed outside a small group of experts.

The IUT theory\(^7\) uses some key theorems in anabelian geometry, as well as its later developments such as absolute anabelian geometry and mono-anabelian geometry. One starting observation for arithmetic deformation theory, i.e. IUT, is that unlike the usual algebraic geometry in which working with schemes corresponds to working with rings, working with certain anabelian objects corresponds to working with topological groups, thus using one operation instead of two, with new options to perform kinds of arithmetic deformation, not available in the standard arithmetic geometry. The total volume of relevant papers in anabelian geometry used in one or another extent in IUT is huge, even though it is possible not to read all of its 1500 pages.

2. **The study of IUT.** Links to various study materials about IUT are available from pages of the author of IUT\(^8\). The total amount of time dedicated to the verification process of IUT by mathematicians already exceeds

\(^4\)appropriate for the given period of development of math

\(^5\)For more details and related math issues see [https://www.maths.nottingham.ac.uk/plp/pmzibf/232.pdf](https://www.maths.nottingham.ac.uk/plp/pmzibf/232.pdf).

\(^6\)In the 1990s, a series of results about anabelian properties of Galois groups of global and higher global fields, i.e. birational anabelian geometry, were obtained by F. Pop. Since the early 1990s, F. Bogomolov suggested and developed, later in collaboration with Yu. Tschinkel, his birational anabelian geometry for varieties of dimension \(\geq 1\) over algebraically closed fields, this theory is quite different from arithmetic anabelian geometry in many respects.

\(^7\)Produced alone, compare: ‘They’ve all done things, often beautiful things, in a context that was already set out before them, which they had no inclination to disturb. Without being aware of it, they’ve remained prisoners of those invisible and despotic circles which delimited the universe of a certain milieu in a given era. To have broken these bounds they would have to rediscover in themselves that capability which was their birth-right, as it was mine: the capacity to be alone.’, pp. 34–35 of the English transl. of A. Grothendieck’s ‘Récoltes et Semaille’, [http://matematicas.unex.es/~navarro/res/lisker1.pdf](http://matematicas.unex.es/~navarro/res/lisker1.pdf).

\(^8\)[http://www.kurims.kyoto-u.ac.jp/~motizuki/top-english.html](http://www.kurims.kyoto-u.ac.jp/~motizuki/top-english.html)
30 researcher-years. This definitely looks to be the largest time ever spent in the history of mathematics on the verification of a mathematical work prior to its publication. Several international conferences were organised in 2014–2016. Numerous intensive seminars have been held in Japan (2012–2019), UK (2015-2018), China (2015-2016), they involved nationals of many countries. In addition to the referees’ comments, active learners of IUT sent in 2012–2017 a 4-digit number of comments, questions, remarks, all had been carefully taken into account by the author. Several learners of IUT shared their understanding of it, by writing texts and surveys. There are already more surveys of IUT than of any previous fundamental work at the time of its publication.9

The absence of experts in anabelian geometry worldwide, the lack of good digestion of Grothendieck’s heritage and the current situation with top research in number theory have substantially affected the ability of contemporaries to study IUT. Some people applied serious efforts to study IUT for some time, but stopped—indeed, the task is huge. IUT can be a difficult theory to study for experts from other standard number theoretical areas, such as arithmetic of elliptic curves, Galois representations and diophantine geometry, since they cannot easily apply their previous expertise without learning first, as PhD students, new for them areas such as arithmetic anabelian geometry. It is crucial to appreciate that whatever are one’s previous results in other areas of number theory, they do not make one an expert in anabelian geometry and IUT. It is sometimes forgotten that the time issue weakens mathematicians’ strength, including the ability to study entirely new theories. Math pioneering power rarely stays with the same mathematician for a long period of time. More researchers in the midst of their career than ever prefer to stay within their narrow area and show no enthusiasm or ability to learn new groundbreaking theories. Mathematicians who can understand standard/conventional things quickly, who can master a domain that has already been well established may have not good chances to progress in the study of IUT, unless they are good to do pioneering things.10

Recommendations to mathematicians who are interested to study IUT. Pathways to study IUT are available from many sources including www-links in footnotes of this text. If you find a piece of IUT looking to you as an error, and you cannot resolve it, document your evidence and contact the author or experts to discuss. Some may be waiting for a ready-to-digest version of the theory but it will take time for it to be produced.11

3. The reaction to IUT.

3.1. On some of reaction to IUT by mathematicians. Who can doubt that any professional consensus about any mathematical theory can only come from experts in its subject area. Mochizuki’s work includes fundamental contributions in numerous directions: Hodge–Arakelov theory, anabelian geometry, mono-anabelian geometry, combinatorial anabelian geometry, Grothendieck–Teichmüller group, p-adic Teichmüller theory, inter-universal Teichmüller theory. Except for the last direction, none of his work has ever been criticised — because it was read and appreciated by experts.

Several expected candidates to study the theory chose to be as unambitious as it can get by doing essentially nothing about its study for seven years. Some preferred to adopt the convenient stance of sceptical attitude not based on any expert knowledge of the subject area. Some chose to spread negative false rumours, sometimes in order to justify their inability to study the theory.12

One of such untrue rumours aims to cause a confusion (and sometimes help to promote some less important work) by talking about some kind of controversy about the status of IUT — however, to have a controversy about a math work there should be genuine experts on both sides of the argument able to provide valid math

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9 For their incomplete list see e.g. http://www.maths.nottingham.ac.uk/plp/pmzibf/guidestoIUT.html.
10 https://www.bbc.co.uk/news/world-europe-50856999
11 Galois theory was ‘notoriously difficult for his contemporaries to understand, especially to the level where they could expand on it’, and its digestion took decades; its best presentation by E. Artin appeared in the next century.
12 See e.g. the report about the Oxford IUT workshop https://www.maths.nottingham.ac.uk/plp/pmzibf/files/iut-i-rep.html
arguments but this is not the case for IUT. Talking negatively about IUT always encounters the glaring problem: the sheer inability to indicate any concrete valid error or ask at least some good questions about IUT.

A naive request to provide more details and explanations for the IUT papers and a psychologically comfortable intention to wait for this to happen demonstrates lack of basic knowledge of the situation: the level of presentation of IUT was already very detailed in 2012, and in the last 7 years the volume of the IUT papers has increased by some 100 pages more, reaching almost 600 pages. In comparison, 10 surveys of IUT, which can appeal to some readers, are 2 or more times shorter.

The first question to ask to any mathematician spreading rumours about something wrong in IUT is about their expertise in anabelian geometry and then for a concrete documented math evidence of any fault in IUT.

3.2. Articles about IUT in mass media of some countries. IUT has attracted a high level of interest from mass media. There are some reasonably good written articles about IUT and its author. At the same time, there are irresponsibly written articles presenting very inaccurate pictures.

Most experts on IUT decline to answer journalists questions, so then journalists contact mathematicians or even laypersons with zero publication record in number theory. Some of the interviewed mathematicians are good in their own areas, but that does not make them experts in areas they do not know. Experience in classical Diophantine geometry, algebraic geometry, modularity, Galois representations or aspects of p-adic geometry does not enable one with the expert intuition and knowledge of anabelian geometry and IUT.

One of standard ways for journalists to write their articles is to present opposite points of view but in the case of IUT the journalists often fail to appreciate that they mix experts opinions (all of which are positive) with negative or ignorant opinions of non-specialists. It is similar to as if an article about the value of a graduate course is written by mixing opinions of its students with grade A and its students who never attended any lecture and got grade F, or asking a farmer about general relativity theory.

Recommendation to serious journalists. Before interviewing a mathematician about IUT, first check the expertise level by asking several simple questions such as their knowledge of and expertise in anabelian geometry, including their publications record, the number of hours spent on the study of IUT and whether they asked questions about IUT to the author of IUT or experts in IUT.

3.3. An attempt to study IUT by two German mathematicians and ethical issues. In 2013–2017 not a single concrete mathematical remark indicating a serious problem in IUT was produced. Since 2014 P. Scholze kept talking publicly about faults in IUT. Eventually Scholze visited RIMS, together with J. Stix, in March 2018, just for 5 days. After the meeting, Scholze and Stix came with their caricature version of IUT based on their oversimplification of IUT in which they identify all isomorphic rings and ‘forget’ about the fundamental role of automorphism groups. In particular, the two German mathematicians deny the use of anabelian geometry and infinitely many theatres in IUT. Initially, Scholze and Stix intended to put their report about the meeting online. However, after reading Mochizuki’s report on their report, see especially its sect. 13 The author of this text and some other people wrote to him several times asking to tell precisely what were the faults in IUT he knew. He declined to participate in IUT workshops. Eventually, he sent just one most loosely stated question to Mochizuki in May 2015, perhaps related to the so called Ind3 indeterminacy. Ind1-Ind3 are three fundamental indeterminacies in IUT one needs to allow in order to have certain functoriality/multiradiality. The author of IUT responded to him with a long email that also offered to conduct discussions via email to address any questions, but Scholze declined to communicate further. Part of this is stated on p.3 of the main Mochizuki’s report http://www.kurims.kyoto-u.ac.jp/~motizuki/Rpt2018.pdf at his page http://www.kurims.kyoto-u.ac.jp/~motizuki/IUTch-discussions-2018-03.html. 14 Mochizuki writes about this on the same p.3, ‘On the other hand, the March 2018 discussions centred around quite different issues, such as (Ind1,2)’. Note: not Ind3 anymore. 15 For a popular presentation to high school students of the importance to use infinitely many theatres in IUT, one can watch F. Kato’s talk https://www.youtube.com/watch?v=fNS7N0DLaQ&vl=en and read his bestselling book https://twitter.com/FumiharuKato.
17-18\textsuperscript{16} and these comments\textsuperscript{17}, they completely changed their mind in July 2018 and stopped to be interested to post their own report. They eventually agreed to let the author of IUT to include their report on his pages. The author of IUT formulated several questions to the German mathematicians in his report that may have helped them to appreciate how erroneous was their take on IUT. The second version of their report did not address most of comments of Mochizuki on their first report. The second version of their report also included new incorrect statements such as a blunder in classical height theory and a fundamental misunderstanding of one of Faltings work. Their short lived study of IUT\textsuperscript{18} stands in shark contrast with the deep study of it by the other mathematicians mentioned above, who asked/made many good questions, remarks and comments.

If one does not apply appropriate efforts to study the area of a fundamentally new theory, one does not become an expert in it, whatever one’s own different area of specialisation is. Of course, it is still possible to contribute useful questions, comments, remarks in relation to more conventional parts of the theory, e.g. those that came in 2012 from two analytic number theorists. To make a mistake in one’s mathematical study is not unusual, especially when one tries to understand a complex theory going much deeper than standard research. But to publicly talk about faults in another theory for several years without ever having any valid evidence of the faults?

The failure of those two German mathematicians in their rushed study of IUT should not stop serious researchers to study IUT and can serve as a good lesson. The failure of Mrs. Lancaster to understand the question does not in any way imply anything negative about the question.

3.4. Abject reaction. In the first approximation, the number of ignorant negative reactions to IUT, not based on valid math knowledge, was inversely proportional to the number of home academicians capable to study the theory. Something is fundamentally rotten here, and it has to be addressed properly. Why did these few mathematicians make public their opinions about a fundamental development in the subject area where they have empty research record, with no evidence of their serious study of it, and without providing any math evidence of errors in the theory? If they feel bad about their own inability to study the theory, it is still not the reason to behave irresponsibly. If one is reckless, one should learn the error of making hasty judgments and come to appreciate the difference between the superficial and the essential. The less innocent reasons are pursuing goals having nothing to do with the theory they chose to say negative things about. There are also questions to some research institutes.

The reader, do you have deja vu in relation to the failure of some mathematicians or math research institutes to address challenges associated with pioneering math research and its study?

4. Developments. Several are mentioned above.

The April 2019 book by F. Kato about IUT presents various features of IUT to the wider audience. This book was in the list of top twenty bestselling books in all subject areas on amazon in Japan, and it was awarded the Yaesu prize\textsuperscript{19}.

There are already new math developments related to IUT, in different directions.

Four international workshops on anabelian geometry and IUT are organised during a special RIMS Project Research year on Expanding Horizons of Inter-universal Teichmüller Theory in 2020–2021\textsuperscript{20}, supported by the new Center for Research in Next-Generation Geometry.

\textsuperscript{16} see footnote 13

\textsuperscript{17} http://www.kurims.kyoto-u.ac.jp/~motizuki/Cmt2018-05.pdf

\textsuperscript{18} In 1931 a group of scientists published a book ‘Hundert Autoren gegen Einstein’. This book is now viewed as ‘a reaction of an inadequately educated academic citizenship, which didn’t know what to do with relativity’ and as an ‘accumulation of naive errors’.

\textsuperscript{19} https://twitter.com/yaesu_paseo/status/1190084381529986721?ref_src=twsrc%5Etfw