ABOUT CERTAIN ASPECTS OF THE STUDY AND DISSEMINATION OF SHINICHI MOCHIZUKI'S IUT THEORY

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This text aims to communicate in a compact form some of factual information related to the study of Shinichi Mochizuki's IUT theory[1] and its dissemination, as well as some related aspects including non-experts behaviour. Some of more general issues are discussed in two other papers[2 3]. Without repeating the content of those papers, this text deals with some concrete issues, rather related to some mathematicians than to mathematics. The summary of what follows is this.

Number theory consists of many different areas and the distance from one area to the rest can be large[4]. To follow new fundamental developments one typically needs to study relevant prerequisites: for Deligne's proof of GRH in positive characteristic one needs to know the work of Grothendieck, for non-abelian developments in the Langlands program one needs to know representation theory, etc. The main prerequisite for IUT is the vast area of (arithmetic) anabelian geometry developed since 1990 in Japan, and the distance from it to most other areas of number theory is large. In 2012 there were few authorities in arithmetic anabelian geometry outside Japan. IUT is an increase of mathematical knowledge in the area of anabelian geometry in which there are relatively few working experts, this is very different from various other developments such as the theorem of Faltings and the theorem of Wiles. To gain a good understanding of IUT, one has to invest an adequate large amount of time in a dedicated serious focused study of the theory starting with anabelian geometry. This cannot be done in relatively short period of time. To help mathematicians to study IUT, a very large amount of time and effort has been dedicated since 2015 to the dissemination of IUT, via various workshops, including large international, via seminars, lectures and study groups. No valid math evidence of any serious fault in IUT, confirmed by professionals, has been found by anyone. Minor oversights have been found and corrected. To this day there remains no mathematically substantive reason whatsoever to doubt the validity of IUT. The number of researchers who have mastered IUT with exceptional thoroughness by investing a large amount of time and effort, is steadily growing. Learners of IUT have sent a 4-digit number of questions and remarks to the author, all addressed. A two-digit number of surveys of IUT and a highly popular book on IUT, by mathematicians from several countries, individually present the theory in different ways. There are more mathematicians able to produce professional reports on the IUT papers than the number of such reports on previous very rare major math breakthrough at the time of their publication.

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[1] The IUT papers made public in August 2012 are available from section Inter-universal Teichmüller Theory of its author page `http://www.kurims.kyoto-u.ac.jp/~motizuki/papers-english.html`. See those pages for various information on seminars and workshops on IUT. See also this page `https://www.maths.nottingham.ac.uk/plp/pmzibf/guidestoiut.html`.

[2] I. Fesenko, Arithmetic deformation theory via arithmetic fundamental groups and nonarchimedean theta functions, notes on the work of Shinichi Mochizuki, Europ. J. Math. (2015) 1:405–440, available from `https://www.maths.nottingham.ac.uk/plp/pmzibf/notesoniut.pdf`

[3] I. Fesenko, Remarks on aspects of modern pioneering mathematical research, available from `https://www.maths.nottingham.ac.uk/plp/pmzibf/rpp.pdf`

[4] this also depends on the stage of development of the area

In 2018–2019 two year long IUT seminars at RIMS for new learners were conducted. 2021 is a special RIMS year with 4 international workshops on anabelian geometry, combinatorial anabelian geometry and IUT.

In view of its fundamental applications, it is natural that number theorists from other areas showed some interest to IUT. Researchers from different countries in the course of several years of their study of IUT have asked interesting or deep questions and contributed to new original developments. Some other researchers have tried to study IUT just for a short while, without mastering anabelian geometry and without attending various workshops on IUT, and failed. A tiny group of people with no knowledge of anabelian geometry were active in publicly making negative absurd remarks about IUT, always devoid of any serious math substance. Few researchers chose to spread fake news and disinformation about IUT. Their behaviour might have affected some people unable to distinguish an expert from a non-expert. Even worse, their irresponsible action might have discouraged or scared some mathematicians to start and continue their long-term work on on key fundamental problems and theories, thus affecting truly novel developments in math.

1. On mathematical environment around IUT, briefly. Class field theory, the heart of algebraic number theory, has several important generalisations. They include the Langlands correspondences, anabelian geometry and higher class field theory. By various reasons the first generalisation[5] has attracted many times more researchers than the second and the third, but all of these generalisations of class field theory are fundamentally important. Most of the central problems in the second and third generalisations of class field theory have been settled[6]. One can imagine another universe where higher class field theory and anabelian geometry attract many more researchers than in this universe and where general class field theory concepts are well understood by all number theorists.

The main prerequisite for IUT theory is arithmetic anabelian geometry, including Mochizuki's famous proofs of the Grothendieck conjecture and his absolute and mono-anabelian geometry. Arithmetic anabelian geometry was started in works of Neukirch–Ikeda–Uchida–Iwasawa for small fields (such as number fields or their completions) in characteristic zero, and from a different motivation for hyperbolic curves over number fields it was proposed by Grothendieck. The main leading country in arithmetic anabelian geometry is Japan, and the first three contributors to anabelian geometry for hyperbolic curves were H. Nakamura, A. Tamagawa and Sh. Mochizuki.[7] Below 'anabelian geometry' will mean 'arithmetic anabelian geometry'. In the last thirty year a vast body of fundamentally important results in anabelian geometry were established. These developments were essentially left unnoticed in many countries and outside a small group of experts. Anabelian geometry is very much different from such other developments as the Langlands correspondence over number fields and the expertise in the latter is almost orthogonal to the required expertise in the former.

The IUT theory uses some key theorems in anabelian geometry, as well as its later developments such as absolute anabelian geometry and mono-anabelian geometry. The total volume of relevant papers in anabelian geometry used in one or another extent in IUT is huge. One starting observation for arithmetic deformation theory, i.e. IUT, is that unlike the usual algebraic geometry in which working with schemes locally corresponds to working with commutative rings, working with certain anabelian objects corresponds to

[5] even though it is still lacking a version parallel to general class field theory, see the next footnote

[6] for more details and related math issues see `https://www.maths.nottingham.ac.uk/plp/pmzibf/232.pdf`

[7] In the 1990s, a series of results about anabelian properties of Galois groups of global and higher global fields, i.e. birational anabelian geometry, were obtained by F. Pop. Since the early 1990s, F. Bogomolov suggested and developed, later in collaboration with Yu. Tschinkel, his birational anabelian geometry for varieties of dimension > 1 over algebraically closed fields, this theory is quite different from arithmetic anabelian geometry in many respects.

working with large nonabelian topological groups, thus using one operation instead of two, with new options to perform kinds of arithmetic deformation, not available in the standard arithmetic geometry. There is an associated fundamental problem to measure the deviation of certain diagrams of groups and maps between groups from being commutative. This problem is solved by IUT in the specific setting of hyperbolic curves related to elliptic curves over number fields, thus eventually providing bounds on certain deformations.

2. The study of IUT. There are several options in relation to perception of any pioneering math work. The best is to study it yourself and base your opinion on your math knowledge of it. The next is to refrain from judgement of the theory when one does not have expertise in the relevant subject area.[8] Another option is to do the same as with previous breakthroughs: believe experts in the theory. For example, most mathematicians 'believe' Deligne's proof of the generalised Riemann hypothesis for varieties over finite fields without ever having checked it themselves.

Links to various study materials about IUT are available from pages of the author of IUT[9]. The total amount of time dedicated to the verification process of IUT by mathematicians is several decades, and it looks to be the largest time ever spent in the history of mathematics on the verification of a mathematical work prior to its publication. Vast opportunities to study IUT have been open since September 2012. It is possible to contribute useful questions, comments, remarks, e.g. in relation to more conventional parts of the theory, e.g. such as those that came in 2012 from classical number theorists.

The IUT papers were published in March 2021.[10] The IUT papers have been checked and verified [11] by (a) a group of appointed referees, for 8 years, with 10 revisions of the original paper; (b) about 20 mathematicians of many nationalities who have sent more than 1000 of their questions and remarks to the author, all answered and taken into account when relevant; (c) a RIMS seminar in 2015 and two international IUT workshops in 2015 and in 2016; (d) two RIMS seminars in 2018/2019 and in 2019/2020; (e) a year long Nottingham–Paris IUT seminar in 2018/2019.

An ongoing 2020-2021 international online seminar on IUT[12] involves researchers from seven countries.

One can occasionally hear a naive request to provide more details and explanations for the IUT papers, with an associated psychologically comfortable attitude to wait for this to happen. This demonstrates the sheer lack of basic knowledge of the situation: the level of presentation of IUT was already very detailed in 2012 and in the last 7.5 years the volume of the IUT papers had increased by some 100 pages more, reaching almost 600 pages, and there are already many surveys of IUT and even a book for the general audience. It is true that it is not easy to read the IUT papers, whose presentation reflects the fact that we are currently missing the right language to describe the theory; however, there are many important papers which are very difficult to read.

Some people applied serious efforts to study IUT for some rather short period of time, without attending IUT workshops and apparently without studying anabelian geometry, but stopped. This is normal in relation

[8] A. Beilinson wrote 'I believe that in mathematics, as everywhere else, you can say that something is correct or not only if you have understood this yourself. Since we do not have time to do everything, in mathematics I tend to believe that something is correct if I can understand some pieces of the proof or theory. If I do not understand anything, I try to refrain from making judgement.' From email correspondence in January 2018.

[9] `http://www.kurims.kyoto-u.ac.jp/~motizuki/top-english.html`

[10] `https://www.ems-ph.org/journals/show_issue.php?issn=0034-5318&vol=57&iss=1`

[11] For an incomplete list of various activities see e.g. `http://www.maths.nottingham.ac.uk/plp/pmzibf/guidestoiut.html`.

[12] `http://www.kurims.kyoto-u.ac.jp/~bcollas/IUT/IUT-schedule.html`

to the study of complex theories. Whatever are one's previous contributions to other areas of number theory, those do not make one an authority in anabelian geometry and IUT. Of course, we would like to see more learners of IUT. Mathematicians can and should help to produce more learners of IUT in their departments, especially if their departments attract most talented young mathematicians in their country.

Small numbers of experts in anabelian geometry, a relatively poor digestion of Grothendieck's 50 years old math heritage by number theorists, a large distance from anabelian geometry and IUT to currently fasionable directions, a large number of new concepts in IUT and its large volume, as well the general situation of substantially weakened research in number theory in many countries have affected the study IUT and its perception.

Recommendations to mathematicians interested to study IUT. Pathways to study IUT are available from many sources including www-links in footnotes of this text. If you find a piece of IUT looking to you as an error and you cannot resolve it, document your evidence and contact the author or its learners to discuss. Some may be waiting for a ready-to-digest version of the theory but it will take time for it to be produced.[13]

3. On negative aspects of reaction to IUT.

3.1. On reaction to IUT from some mathematicians. Mochizuki's work includes fundamental contributions in numerous directions: Hodge–Arakelov theory, anabelian geometry, mono-anabelian geometry, combinatorial anabelian geometry, Grothendieck–Teichmüller group, p-adic Teichmüller theory, inter-universal Teichmüller theory. Except for the last direction, none of his work has ever been criticised because it was read and appreciated by experts in the subject area. 'Love of knowledge, without a love to learn, finds itself obscured by loose speculation'.[14]

'You can lead a horse to water but you can't make her drink'. Few mathematicians chose to talk in a benighted way about IUT and its study, while being fully aware they simply do not have any authority in the subject area. Talking exclusively with non-experts, who have very weird ideas about IUT, can only produce weird outcomes. They made public their ignorant negative opinions about a fundamental development in the subject area where they have empty research record, with no evidence of their serious study of it, and without providing any math evidence of errors in the theory.[15] Non-expert negative opinions about IUT seeded a pernicious mistrust of this rare breakthrough and pioneering math research in general. Their behaviour contributes to the erosion of professional norms.

In particular, there are no active US researchers in anabelian geometry of hyperbolic curves over number fields, but most of irrational negative comments about IUT originated from a tiny group of mathematicians in that country.

Some chose to spread a malicious distortion of the math truth or false rumours. One of them is talking about some kind of controversy about the status of IUT. This is not an argument that can hope to be accepted: in order to have a controversy about a mathematical work there should be genuine experts on both sides of the argument able to provide valid math arguments which can pass peer review. This is plainly not the case for IUT: not a single expert in IUT is known who sees mistakes in the published version of IUT and none of internet critical remarks about IUT can pass peer review. This also explains why not the trial of serious math peer review but the choice of shallow posting is the only venue for non-expert public chats about IUT.

[13] Galois theory was 'notoriously difficult for his contemporaries to understand, especially to the level where they could expand on it', and its digestion took decades; its best presentation by E. Artin appeared many decades later.

[14] Confucius, The Analects

[15] See e.g. the report about the Oxford IUT workshop `https://www.maths.nottingham.ac.uk/plp/pmzibf/files/iut-i-rep.html`

There is only one professional side, the side of experts in IUT, which includes many those who have worked for years to learn the subject area and the theory. They, together with the referees and the group of editors processing the IUT papers, have all concluded that the IUT papers have no mathematical flaws. Part of this process was a truly unprecedented event when the author of IUT kept investing a lot of time in answering more than 1000 of questions for more than 7 years.[16]

We could not fail to be deeply concerned about several exhibited biases in relation to the study of IUT. They include applying double standards with respect to who can be counted as a legitimate independent referee/reviewer of research work, about how young researchers in different countries ask questions, and believing that Western math journals are superior to math journals elsewhere with respect to processing submitted papers. These biases should be strongly rejected.

3.2. Some articles about IUT in mass media. IUT has attracted a high level of interest from mass media. Most experts in IUT decline to answer journalists questions, so then journalists contact mathematicians who are not experts in anabelian geometry or even laypersons with zero publication record in number theory. Some articles cite opinions of non-experts only, mathematicians with zero track record in anabelian geometry. Experience in areas such as classical Diophantine geometry, algebraic geometry, modularity, Galois representations or aspects of p-adic geometry does not enable one with the required intuition and knowledge of anabelian geometry and IUT.

One of easiest ways for journalists to write their articles is to present opposite points of view but in the case of IUT the journalists often fail to appreciate that they mix experts opinions (all of which are positive) with ignorant opinions of non-specialists who did not want to be kept in the loop in relation to the study of IUT.

Recommendation to serious journalists. Before interviewing a mathematician about IUT, first ask several simple questions such as their knowledge of and expertise in anabelian geometry, delivered talks on anabelian geometry at international conferences, the number of hours spent on the study of IUT and whether they asked questions about IUT to the author of IUT or experts in IUT.

3.3. A short-term attempt to study IUT. In 2013–2017 not a single concrete mathematical remark indicating any essential issue in IUT was produced. A German mathematician Scholze, with a record of work in other areas but with no publications or expertise in anabelian geometry, kept talking publicly about faults in IUT since 2014 without ever providing any math evidence.[17] By and by, after a lot of pressure, he visited RIMS, together with Stix, in March 2018, just for several days. They were asked to produce a report about their study of IUT so that any mathematician can read it. Their first report includes an incorrect version of IUT, based on a gross erroneous oversimplification of IUT in which they identify all isomorphic rings and ‘forget’ about the fundamental role of automorphism groups in anabelian geometry. One cannot be surprised that they deduced that their fake version of IUT was incorrect. Their text includes no proved theorem that their caricature version of IUT equals to IUT, and no such theorem is possible. Their report

[16] compare this intensive study and verification with the next section material

[17] The author of this text wrote to Scholze several times asking to behave professionally and in particular to tell precisely what were the faults in IUT he knew about and discuss with experts, but no response had come. Eventually, Scholze sent just one most loosely stated question to Mochizuki in May 2015. The author of IUT responded to him with a long email that also offered to conduct discussions via email to address any questions, but Scholze declined to communicate further. Part of this is stated on p.3 of the main Mochizuki’s report, see footnote 19.

essentially denies the use of anabelian geometry and infinitely many theatres in IUT[18]. For various details see this report[19] of the author of IUT. The German mathematicians intended to make their report available online, however, after reading the comprehensive report[20] of the author of IUT on their report and these comments[21], they changed their mind and abandoned plans to post their own report. In his comprehensive report on their report the author of IUT formulated few questions to the German mathematicians which may have helped them to appreciate their mistakes.[22] However, the second version of their report failed to address those few questions. Moreover, it included new incorrect statements demonstrating inadequate knowledge of more classical areas such as a blunder in height theory and a fundamental misunderstanding of one of the Faltings work, and those rather mistakes can be easily seen by researchers not familiar with anabelian geometry. Scholze unilaterally withdrew from any further correspondence or study of IUT. This rushed study of IUT simply can not pass any careful peer review process. The author of IUT had to include their reports on his pages, so that any researcher can directly check their numerous flaws.[23] That 'study' of IUT by the two mathematicians[24] stands in shark contrast with many months study of it by a two-digit number of other researchers who, as most serious mathematicians, do not use blogs to express their knowledge and opinions. It is not unusual to make a mistake in one's mathematical study, especially when one tries to understand a complex theory, but to publicly talk about faults in another theory for several years without ever having any valid evidence is culpable and irresponsible and the reluctance to acknowledge one's mistakes is reckless. To stop misleading other mathematicians and behave responsibly with respect to mathematical community, these two mathematicians now have two options: either to publicly acknowledge their mistake or to write a regular paper explaining their take on IUT, with full proofs, and submit it to a journal with high quality of peer review and see the outcome of such review.

4. Developments. A book[25] by F. Kato, published in April 2019, presents various features of IUT to the wider audience. This book was in the list of top twenty bestselling books in all subject areas on amazon in Japan, and it was awarded the Yaesu prize[26].

Learners of IUT can participate in four international workshops on anabelian geometry and IUT are organised during a special RIMS Project Research year on Expanding Horizons of Inter-universal Teichmüller Theory in 2021[27], supported by the new Center for Research in Next-Generation Geometry.

The author of IUT is available to talk with any mathematician who has seriously studied IUT and has questions to ask about it.

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[18] For a popular presentation to high school students of the importance to use infinitely many theatres in IUT, one can watch F. Kato's talk `https://www.youtube.com/watch?v=fNS7NO4DLAQ&vl=en`

[19] the main report `http://www.kurims.kyoto-u.ac.jp/~motizuki/Rpt2018.pdf` at the page `http://www.kurims.kyoto-u.ac.jp/~motizuki/IUTch-discussions-2018-03.html`.

[20] referred to in footnote 19, see especially its §2 and §5

[21] `http://www.kurims.kyoto-u.ac.jp/~motizuki/Cmt2018-05.pdf`

[22] see also Remarks 3.11.1 and 3.12.2 of IUT-III

[23] Putting their report on the page of the author of IUT does not imply in any way its validity, of course.

[24] Compare with the content of published in 1931 book 'Hundert Autoren gegen Einstein' characterised as 'a reaction of an inadequately educated academic citizenship, which didn't know what to do with relativity'and as an 'accumulation of naive errors', at least its authors cared to publish their opinions.

[25] `https://twitter.com/FumiharuKato`

[26] `https://twitter.com/yaesu_paseo/status/1190084381529886721?ref_src=twsrc%5Etfw`

[27] `http://www.kurims.kyoto-u.ac.jp/kyoten/en/index.html`, `http://www.kurims.kyoto-u.ac.jp/~motizuki/project-2021-english.html`