

ABOUT CERTAIN ASPECTS OF THE STUDY AND DISSEMINATION OF SHINICHI MOCHIZUKI'S IUT THEORY

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This text communicates in compact form some of factual information related to the study of Shinichi Mochizuki's IUT theory¹ and its dissemination, as well as various aspects of the situation around IUT. IUT was made public in 2012. It is an extraordinarily novel and fundamentally important development of the 21st century. At the same time, like litmus paper, reaction to its study explicitly uncovers several problems. These and more general issues are discussed in three other papers^{2,3,4}. Without repeating the content of those papers, this text deals with some mundane issues. It also includes some recommendations.

In summary of what is discussed below, we have the following facts.

Fact 1. The main compulsory prerequisite for IUT is the huge area of (arithmetic) anabelian geometry developed in 1990–2014 in Japan. One cannot expect to understand IUT without good knowledge of anabelian geometry. To become an expert in IUT, one has to invest a huge amount of time. There is a 2-digit number of pundits in IUT in 2018, of 10 nationalities, who sent more than 1000 questions and remarks to the author. A very large amount of time and effort has been dedicated to the study and dissemination of IUT. Experts in IUT have not made public negative remarks about it, they do not participate in internet discussions on IUT, and almost none among them gives interviews. And IUT has proved to be sufficiently robust to allow its further extensions.

Fact 2. No valid concrete math evidence of any serious fault in IUT has been supplied by anyone. There is a small group of people, who made public some negative baseless remarks about IUT. For years, none of such comments is derived from any concrete math evidence or from expert knowledge of the relevant area. All these self-proclaimed critics (almost all based in USA) have empty research track record in anabelian geometry and have not demonstrated any evidence of acquired expertise in it. Thus, one is led to believe there are other reasons for their behaviour, having nothing to do with mathematics of IUT. Why did this small group of people need to say something negative about IUT without any serious study of it? See below.

Fact 3. Mass media articles (in English) about IUT sometimes present distorted pictures; the main responsibility for this failure lies on those mathematicians talking to journalists about the subject they do not know.

1. On mathematical environment around IUT, briefly. The main prerequisite for IUT theory is arithmetic anabelian geometry, i.e. anabelian geometry of hyperbolic curves over small fields in characteristic 0 such as number fields or their completions. The main leading centre in arithmetic anabelian geometry is Japan, the main contributors and developers are H. Nakamura, A. Tamagawa and Sh. Mochizuki and other mathematicians. Since the early 1990s, F. Bogomolov suggested and developed, later in collaboration with Yu. Tschinkel, his

¹ The IUT papers are available from section Inter-universal Teichmüller Theory of its author page <http://www.kurims.kyoto-u.ac.jp/~mochizuki/papers-english.html>. See those pages for various information on seminars and workshops on IUT. See also this page <https://www.maths.nottingham.ac.uk/plp/pmzibf/guidestoiut.html>.

² I. Fesenko, Arithmetic deformation theory via arithmetic fundamental groups and nonarchimedean theta functions, notes on the work of Shinichi Mochizuki, *Europ. J. Math.* (2015) 1:405–440, available from <https://www.maths.nottingham.ac.uk/plp/pmzibf/notesoniut.pdf>

³ I. Fesenko, Remarks on aspects of modern pioneering mathematical research, available from <https://www.maths.nottingham.ac.uk/plp/pmzibf/rapm.pdf>

⁴ I. Fesenko, Fukugen, *Inference: International Review of Science* 2 no. 3 (2016), <https://inference-review.com/article/fukugen>

anabelian geometry for varieties over algebraically closed fields, this theory is very different from arithmetic anabelian geometry. Also, since the early 1990s, a series of results about anabelian properties of Galois groups of global and higher global fields, i.e. function fields anabelian geometry, were obtained by F. Pop.⁵

Below 'anabelian geometry' will mean 'arithmetic anabelian geometry'. In the period of approximately 1990–2014 many fundamentally important developments and results in anabelian geometry were conducted and established, all the main conjectures were proved. At the same time, all these developments were essentially left unnoticed by mathematicians in most mathematically prominent countries.

It is crucial to appreciate that there are several main generalisations of class field theory. They include the Langlands correspondences, anabelian geometry and higher class field theory. By historical reasons the first has attracted many more researchers than the second and the third, but all of these generalisations of class field theory are equally fundamentally important. So far, the first generalisation has used only analogies of special class field theory, while the second and third have used general class field theory.⁶

IUT uses various key theorems in anabelian geometry and its later developments such as absolute anabelian geometry and mono-anabelian geometry. The total volume of relevant papers in anabelian geometry used in one or another extent in IUT is huge, even though it is possible to concentrate on some key theorems and not read all of 1500 pages or so. The absence of experts in anabelian geometry worldwide has substantially affected the reaction to IUT and the ability to study it.

2. The study of IUT. Links to various useful materials about IUT are available from pages of the author of IUT⁷. IUT is a difficult theory, but there are many difficult theories. What is correct is that IUT is much more culturally distant from previous theories than any other theory in pure mathematics. This may make it more difficult to study for mathematicians who work in their areas for many years. To study IUT for one week or for two months cannot result in a good knowledge of it.

The total amount of time dedicated to verification of IUT by mathematicians has probably exceeded 30 researcher-years. This seems to be the largest time ever spent in the history of mathematics on the verification of mathematical work prior to its publication. Several international conferences were organised in 2014–2016. Numerous intensive seminars have been held in Japan (2012–2018), UK (2015–2018), China (2015–2016), they also involved nationals of 6 other countries. In addition to referees' comments, many active learners of IUT sent in 2012–2017 several hundreds, if not a four-digit number, of comments/questions/remarks, all had been carefully taken into account by the author. There is a two-digit number of surveys of the theory, their list and other relevant information is available from this page⁸.

At the same time, smaller numbers of mathematicians than expected are known to have applied appropriate efforts to study IUT, and in some countries their number is zero. Several potential candidates to study the theory chose to be as unambitious as it can get, and to do essentially nothing for six years. Some preferred to adopt the stance of sceptical attitude, which however is not based on any expert knowledge of the subject area.

⁵ I learned about various developments in anabelian geometry from its active researchers for many years: in 1991 talking with J. Neukirch, the pioneer in the classical anabelian geometry, with F. Pop in Heidelberg in 1994, with H. Nakamura at IAS in 1996, and with A. Tamagawa, and in 2003–2004 hosting J. Stix as a postdoc, also talking with F. Bogomolov about his work. Bogomolov's earlier proof of the geometric Szpiro inequality (part of it, as was indicated by K. Kremnitzer, was already known in geometry as the Milnor–Wood inequality) has several remarkable similarities to IUT. Since 2013 I studied all anabelian geometry papers mentioned in the text of footnote 2.

⁶ For more detail see <https://www.maths.nottingham.ac.uk/plp/pmzibf/232.pdf>.

⁷ <http://www.kurims.kyoto-u.ac.jp/~motizuki/top-english.html>

⁸ <http://www.maths.nottingham.ac.uk/plp/omzibf/guidestoiut.html>.

Recommendation to mathematicians who are interested to study IUT: 'We must study the whole work, get to know it inside and then draw our conclusions'.⁹ You can use pathways how to study, available from many sources including www-links in footnotes of this text. Anabelian geometry is the fundamental prerequisite for IUT, there will be an anabelian geometry workshop in May 2020.¹⁰ If you find any place which looks to you as an error, and you cannot resolve it, document your evidence and contact the author or experts to discuss.

3. The reaction to IUT.

3.1. The reaction to IUT by mathematicians. A genuine consensus about any mathematical theory can only come from experts in its subject area. Sh. Mochizuki's work includes fundamental pioneering contributions in numerous directions: Hodge–Arakelov theory, anabelian geometry, mono-anabelian geometry, combinatorial anabelian geometry, Grothendieck–Teichmüller group, p-adic Teichmüller theory, inter-universal Teichmüller theory. Except for the last direction, none of his work has ever been criticised — because it was read and appreciated by experts, while all critical (and very vague) comments on IUT are produced by mathematicians far from having any expertise in relevant subjects. No experts have made negative remarks about IUT.

A. Beilinson, the 2018 Wolf Prize winner, wrote 'I believe that in mathematics, as everywhere else, you can say that something is correct or not only if you have understood this yourself. Since we do not have time to do everything, in mathematics I tend to believe that something is correct if I can understand some pieces of the proof or theory. If I do not understand anything, I try to refrain from making judgement.'¹¹ This attitude is shared by many mathematicians, and the vast majority of mathematicians does not find it reasonable to say anything about anabelian geometry and IUT, since they have not studied these subjects. See also 3.4 and 3.5.

3.2. Articles about IUT in mass media. Among all math work of this century IUT has attracted the highest level of interest from mass media. There are well written articles about IUT and its author in mass media, they winnow out the truth. At the same time, there are articles presenting a very distorted picture.

Most experts on IUT decline to answer journalists questions, so then journalists contact mathematicians or even laypersons who have ever said something on the internet about IUT. Some of the interviewed mathematicians are reasonably good in their own areas, but that does not make them experts in the relevant for IUT area of anabelian geometry. Having some experience of work in classical Diophantine geometry, algebraic geometry, modularity, Galois representations or aspects of p-adic geometry does not enable one with expert intuition and knowledge of anabelian geometry and IUT.

One of standard ways for journalists to write their articles is to present opposite points of view but in the case of IUT the journalists often fail to appreciate that they mix experts opinions (all of which are positive) with negative or ignorant opinions of non-specialists. It is similar to as if an article about the true value of a graduate course is written by mixing opinions of its students with grade A and its students with grade F, or even without opinions of students with grade A at all.

An example of recent absolutely ignorant article is in a very strange journal called quanta. Its recent article about IUT includes dozens of incorrect statements and consists almost entirely of opinions of people with no knowledge of the situation and with empty research track record in the relevant areas. To make things more prominent, the author of the article and the main editor had been failed to take them into account two dozens of corrections.

Recommendation to journalists: before interviewing a mathematician about IUT, first check the expertise level by asking several simple questions such as their knowledge of and expertise in anabelian geometry, including peer reviewer publications, the number of hours spent on the study of IUT (less than 200-300 automatically

⁹ from Glenn Gould and Leonard Bernstein: Bach's Keyboard Concerto No. 1 in D minor, from 3:40 https://www.youtube.com/watch?v=9ZX_XCYokQo

¹⁰ <https://www.maths.nottingham.ac.uk/plp/pmzibf/files/May2020.html>

¹¹ personal communication, January 2018

implies the lack of expertise), how many questions about IUT and related prerequisites they have asked to the author of IUT or experts in IUT. For comparison: twenty or more diligent learners of IUT in 2012–2018 have asked or made in total more than 1,000 questions and remarks, none has posted anything critical about IUT.

3.3. The reaction to IUT on the internet. Many mathematicians naively believe everything they read on the internet. Some posts about IUT may lead their readers to believe things which are completely opposite to the truth. It is important to be aware that there is a very small but an active group of bloggers who spread incorrect information or even disinformation about IUT, thus misleading other people. Certain bloggers void of understanding and working in areas far from pure mathematics or number theory or the subject area of IUT, were keen to attract attention to themselves by publishing ignorant or absurd posts about the theory and its study. The way the internet can be exploited by some bloggers creates a major problem for an objective presentation of various aspects of modern mathematics and developments in it. An aggressive coordinated internet campaign run by a small number of people can be quite influential on those who still tend to believe what they read on the internet about mathematics and make them believe to things opposite to the truth.

3.4. Attempt to study IUT by two German mathematicians. In 2013–2017 no concrete mathematical remarks originated from mathematicians making negative public remarks about IUT. The only pseudo-concrete remarks, and without proofs, were made in 2018 by Scholze and Stix. What they criticised was not IUT, but rather their own caricature of it.¹²

Since 2014 Scholze has been talking publicly at various workshops about 'faults' in IUT. I and probably other people did write to him several times asking to tell precisely what were the 'faults' in IUT he 'knew'. He declined to participate in our 2015 Oxford IUT workshop. Eventually, he sent just one very vaguely and broadly stated question to Mochizuki in May 2015, perhaps related to Ind 3. Ind 1,2,3 are three fundamental indeterminacies one needs to allow in order to have certain functoriality/multiradiality in IUT. The author of IUT responded to him with a long email which also offered to conduct discussions via email to address any questions, but Scholze declined to communicate further. Part of this is stated on p.3 of the main Mochizuki's report¹³ at his new page¹⁴. Regrettably, Scholze continued to publicly downgrade IUT. My invitation to visit and discuss was declined. Eventually he visited RIMS in March 2018 just for 5 days. Mochizuki writes about this further on the same p.3, 'On the other hand, the March 2018 discussions centred around quite different issues, such as (Ind 1,2)'. By the end of the visit he and Stix came with their own very strange version of IUT based on their incorrect oversimplification of IUT where they identify all isomorphic rings with one another. In particular, the position of the two German mathematicians weirdly denies the use of anabelian geometry in IUT! The reaction of experts to this oversimplification can be read in sect.18 of the report¹⁵. Their identification of isomorphic rings strikes as incorrect even experts in other areas, e.g. category theorists.

Initially, Scholze and Stix were keen to put their report about the meeting online. However, after reading Mochizuki's reports, see especially its sect. 17-18¹⁶ and these comments¹⁷, they completely changed their mind in July and stopped to be interested to post their own report. They eventually agreed to let the author of IUT to include their report on his pages. The author of IUT stated several questions to the German mathematicians, which may help them to appreciate how erroneous is their take on IUT. The second version of their report does not address most of comments of Mochizuki on their first report, thus showing their lack of interest and

¹² Compare with 'Hundert Autoren gegen Einstein', https://en.wikipedia.org/wiki/Criticism_of_the_theory_of_relativity

¹³ <http://www.kurims.kyoto-u.ac.jp/~motizuki/Rpt2018.pdf>

¹⁴ <http://www.kurims.kyoto-u.ac.jp/~motizuki/IUTch-discussions-2018-03.html>

¹⁵ see footnote 13

¹⁶ see footnote 13

¹⁷ <http://www.kurims.kyoto-u.ac.jp/~motizuki/Cmt2018-05.pdf>

failure to study IUT properly. The second version also includes new incorrect statements such as a blunder in classical height theory and a fundamental misunderstanding of the famous work of the German mathematician G. Faltings.

This failed attempt to 'study' IUT stands in stark contrast with the highly valuable multi-months study of it by the 2-digit number of mathematicians mentioned above. There is a big difference between a belief that is false and one that is delusional. We shall see to which category does this belief belong to.

When one does not apply appropriate efforts to study the area of a fundamentally new theory, one does not become an expert in it, whatever one's own different area of specialisation is and achievements in it. It is irresponsible to make one's general negative opinion about math work public when it is not based on its good knowledge. Of course, it is still possible to contribute useful questions/comments/remarks in relation to more conventional parts of the theory.

Mochizuki's work in anabelian geometry has never been criticised, and a major part of IUT is some advanced anabelian geometry. Without knowing and feeling anabelian geometry, one cannot progress with the study of IUT. It is not a problem to make a mistake in one's mathematical study, especially when one tries to understand a complex theory. However, publicly talking about 'faults' in another theory for such a long time without having any valid evidence of the 'faults' is ungraceful, and so is the absence of visible efforts to answer to the comments of the author of IUT and to acknowledge one's mistakes.

Recommendation to the two German mathematicians: you are welcome to return to the study of IUT. Potentially, you can master it, with a serious attitude to its study. If you do not have time to study it, encourage younger German researchers to study IUT named after the German mathematician O. Teichmüller.

3.5. Ignorant public criticism of IUT. Unusually for mathematical developments, some mathematicians have publicly criticised IUT and its study without having applied any noticeable serious efforts to learn IUT. In the first approximation, the number of negative (not based on expert math opinion) reactions to IUT was inversely proportional to the number of home academicians capable to study the theory.

Ignorant criticism of IUT originates from a very small group of researchers, almost all working in USA, who regularly meet and know each other, as well as Scholze, well. None among these pseudo-critics has published research papers in anabelian geometry, and no evidence of expertise in anabelian geometry has been demonstrated by them. While experts, who have asked 1000 of math questions, were not interested in online cheap talk, negative online criticism went always in a very vague form without any single valid concrete mathematical evidence of any fault in IUT. Sometimes it was hostile to the author of IUT and mathematicians studying IUT.

There are serious questions to these people about their actions. What are the origins and reasons of very irresponsible and disrespectful public behaviour towards the theory, the author and the experts in IUT? Why did these very few mathematicians rush to the wrong judgment on a completely false basis, why do they decide to make public their opinions about a fundamental development in the subject area where they have empty research record, with no evidence of their serious study of it, and without providing any math evidence of errors in the theory? There is something rotten there. When grown up mathematicians publicly negatively talk about another subject area without providing any single supporting evidence, there might be several reasons for that. One is to be reckless, in which case they should learn the error of making hasty judgments and come to appreciate the difference between the superficial and the essential. Do the real reasons of their behaviour, not at all related to mathematics of IUT, aim to pursue goals having nothing to do with the theory they criticise?

4. Perspectives. Some of them are mentioned in the text referred to in footnote 3. A forthcoming book by F. Kato about IUT and its author will provide more general information about various remarkable features of IUT to the wider audience. More workshops on IUT, including several during a special RIMS year "Expanding Horizons of Inter-universal Teichmüller Theory" in 2020–2021, will help to educate more mathematicians. There will be many new interesting developments around IUT.