ABOUT CERTAIN ASPECTS OF THE STUDY AND DISSEMINATION OF SHINICHI MOCHIZUKI'S IUT THEORY

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The aim if this text is to communicate in a compact form some factual information related to the study of Shinichi Mochizuki's IUT theory, first made public in 2012, and its dissemination, as well as various aspects of the situation around IUT. IUT is an extraordinarily novel, interesting, important and unexpected development in mathematics of the 21st century. A more comprehensive presentation of various more general issues is contained in papers[1] [2]. Without repeating the content of those papers, here I include other facts, in particular those which came in recent conversations with mathematicians. In summary, the key facts are

Fact 1. No valid concrete math evidence of any serious fault in IUT has been given so far by anyone. Moreover, IUT is so robust that it allows its further stronger extensions.

Fact 2. To become in expert in IUT, one has to invest something like 2 years of continuous work. There is a 2-digit number of pundits in IUT in 2018, from 6 countries. Time dedicated to the work leading to IUT and to its study by others exceeds 50 years. Hundreds of questions about IUT have been answered, they included many interesting comments and remarks. In terms of its mathematical value and invested effort, this valuable activity stands in stark contrast with the study of IUT by Scholze and Stix whose attempt to demonstrate a possible 'fault' in IUT was unsuccessful.

Fact 3. The main compulsory prerequisite for IUT is arithmetic anabelian geometry developed in 1990–2014 in Japan. There were/are no active experts in arithmetic anabelian geometry or IUT in 2012/2018 in several mathematically prominent countries including USA. Experts in IUT have not made public negative remarks about it. They do not participate in internet discussions on IUT and almost none among them gives interviews.

Fact 4. Almost all negative remarks about IUT come from a very small group of people working in areas far from anabelian geometry, their associates, or even from people with no arithmetic geometry track record. The authors of those opinions often behave irresponsibly by talking about mathematics they do not know and by misleading other people who cannot distinguish an expert from a non-specialist. Some internet texts about IUT are so incorrect that they can induce opinions which are antipode to the truth. Mass media articles about IUT typically mix expert opinions with those of people who have empty research track record in the subject area, and these articles fail to inform about the expertise of people they quote, thus misinforming their readers.

1. On mathematical environment around IUT briefly. The most important prerequisite for IUT theory is arithmetic anabelian geometry, i.e. anabelian geometry of hyperbolic curves over small fields in characteristic 0 such as number fields or their completions. Below 'anabelian geometry' will mean 'arithmetic anabelian geometry'. The main leading centre in anabelian geometry is Japan, the main contributors and developers are H. Nakamura, A. Tamagawa and Sh. Mochizuki and other mathematicians. Since the early 1990s, F. Bogomolov suggested and developed his anabelian geometry for varieties over algebraically closed fields, this theory is very different from arithmetic anabelian geometry. Also, since the early 1990s, a series of results about anabelian

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[1] I. Fesenko, Arithmetic deformation theory via arithmetic fundamental groups and nonarchimedean theta functions, notes on the work of Shinichi Mochizuki, Europ. J. Math. (2015) 1:405–440, available from `https://www.maths.nottingham.ac.uk/plp/pmzibf/notesoniut.pdf`

[2] I. Fesenko, Remarks on aspects of modern pioneering mathematical research, available from `https://www.maths.nottingham.ac.uk/plp/pmzibf/rapm.pdf`

properties of Galois groups of global and higher global fields, i.e. function fields anabelian geometry, were obtained by F. Pop, but he has been working on Bogomolov's anabelian geometry in the last decade.

In the period of approximately 1990–2014 many fundamentally important developments and results in anabelian geometry were conducted and established, all the main conjectures were proved. At the same time, all these developments were essentially left unnoticed by mathematicians in most mathematically prominent countries. It is crucial to appreciate that there are several main generalisations of class field theory. They include the Langlands Program (where so far only analogies of special class field theory have been used for successful proofs), anabelian geometry and higher class field theory. By historical reasons the first has attracted many more times of researchers than the second and the third, but all of these generalisations of class field theory are equally fundamentally important.[3] I learned about various developments in anabelian geometry from its active researchers for many years. In 1991 I talked with J. Neukirch, the pioneer in the classical anabelian geometry, i.e. for number fields, then talked with F. Pop in Heidelberg in 1994 and then with H. Nakamura at IAS in 1996, then participated in a European network on Galois theory and explicit methods and its conferences on topics related to anabelian geometry, attending talks of A. Tamagawa and talking with him, also hosting J. Stix in 2003-2004 as a postdoc, studying all anabelian geometry papers mentioned in the text of footnote 1, and in recent years talking with F. Bogomolov. Bogomolov's earlier proof (part of it, as was indicated by K. Kremnitzer, was already known in geometry as the Milnor–Wood inequality) of the geometric Szpiro inequality has several remarkable similarities to IUT, see footnote 1, this text[4] and pp.35–37 of the text of footnote 8.

IUT uses various key theorems in anabelian geometry and its later developments such as absolute anabelian geometry and mono-anabelian geometry. The total volume of relevant papers in anabelian geometry used in one or another extent in IUT is huge, even though it is possible to concentrate on some key theorems and not read all of 1500 pages or so. The absence of experts in anabelian geometry worldwide has substantially affected the reaction to IUT and the ability to study it. In 2012–2018 in most countries including USA there are no active researchers who know well arithmetic anabelian geometry. M. Saidi in the UK collaborates with A. Tamagawa and few people in Paris know some anabelian geometry, and in Germany J. Stix did some work in anabelian geometry in positive characteristic (which is quite different from arithmetic anabelian geometry, for example no cyclotomic rigidity issues, so central for arithmetic anabelian geometry, show up in positive characteristic).

2. The study of IUT. IUT is a difficult theory, but there are so many difficult theories. What is correct is that IUT is much more culturally distant from previous theories than any other theory in pure mathematics. This may make it more difficult to study for mathematicians who work in their areas for many years.

Eight mathematicians have prepared a joint short statement to be released when the IUT papers are published, they include researchers of five nationalities. The number of people who can confirm there are no mistakes in IUT is 2-digital. All people who confirm there are no mistakes confirm that not because some other people had done the same, but because they have studied the theory themselves, patiently and diligently, for at least two years, asking many questions. To study it for one week or for two months is simply not enough. Several bright young researchers in several countries joined the study. It is clear that future experts on IUT will be young researchers (PhD or postdoc level). Hence running periodic activities on IUT and anabelian geometry is useful. RIMS will run its special IUT year in 2020. Links to various useful materials about IUT are available from pages of the author of IUT[5] and also from this page[6].

[3] https://www.maths.nottingham.ac.uk/plp/pmzibf/232.pdf

[4] Sh. Mochizuki, Bogomolov's proof of the geometric version of the Szpiro conjecture from the point of view of inter-universal Teichmüller theory, Res. Math. Sci. 3(2016), 3:6, available from http://www.kurims.kyoto-u.ac.jp/~motizuki/Bogomolov%20from%20the%20Point%20of%20View%20of%20Inter-universal%20Teichmuller%20Theory.pdf

[5] http://www.kurims.kyoto-u.ac.jp/~motizuki/top-english.html

[6] https://www.maths.nottingham.ac.uk/plp/pmzibf/guidestoiut.html

Much smaller numbers of mathematicians than expected are known to have applied appropriate efforts to study IUT. Various candidates to study the theory chose to do essentially nothing for six years or to adopt the stance of sceptical attitude not based on expert knowledge of the subject area. Unusually for mathematical developments, some mathematicians felt appropriate to publicly criticise IUT and its study without having applied any serious efforts to learn IUT. In the first approximation, the number of negative reactions to IUT was inversely proportional to the number of home academicians capable to study the theory. While experts were not interested in online cheap talk, negative online criticism went always in a very vague form without any single valid concrete mathematical evidence of any fault in IUT. Sometimes it was hostile to the author of IUT and mathematicians studying IUT. Some of negative posts were apparently coordinated in order to achieve goals having nothing to do with IUT. Certain media, as well as few bloggers void of understanding and working in areas far from number theory or the subject area of IUT, were keen to attract attention to themselves by publishing ignorant or absurd articles and posts about the theory and its study.

3. The reaction to IUT.

3.1. The reaction to IUT in mass media. It is likely that no other work in pure mathematics in the 21st century has attracted so much attention of mass media. Almost all experts on IUT decline to answer journalists questions, so then journalists contact laypersons who have ever written something on the internet about IUT. Some of them are good mathematicians in their own areas, but of course that does not make them experts in anabelian geometry or IUT, similarly to how most of number theorists are not experts in its interaction with K-theory. Having some experience of dealing with such classical issues as modularity, Galois representations or aspects of p-adic geometry viewed classically enables one with no intuition or knowledge in anabelian geometry and IUT, which do not use representation theory and have nothing to do with modularity. One of standard ways for journalists to write their articles is by trying to present opposite points of view but in the case of IUT the journalists do not quite understand that they mix experts opinions (all of which are positive) with negative or ignorant opinions of non-specialists. It is similar to as if an article about the true value of a graduate course is written by mixing opinions of its students with grade A and its students with grade F, or even without opinions of students with grade A at all.

3.2. The reaction to IUT on the internet. No other math work has attracted so much of fundamentally incorrect representation on the internet as IUT. If one reads texts about IUT on the internet, one may be led to believe things which are completely opposite to the truth. Many mathematicians naively believe what they read on the internet. There is a very small but unusually active group of bloggers, mathematicians and their associates including non-number theorists, who spread incorrect information or even disinformation about IUT, thus misleading many other people, see 3.4. At the same time, experts in IUT do not participate in vague internet discussions (and of course they are not obliged to), so their voice is essentially not represented on the internet.

Indeed, more generally, the way the internet may be used by some bloggers creates a major problem for an objective presentation of various aspects of modern mathematics and developments in it. An aggressive coordinated internet campaign run by a small number of people can be quite influential on those who still tend to believe what they read on the internet about mathematics and make them believe to things opposite to the truth. Certain internet sites with math questions and answers include too many incorrect answers, thus potentially making more harm than use for people reading them. One lesson is that currently one simply should not believe any math information on the internet which is not supported by peer reviews or expert opinions.

A. Beilinson, the 2018 Wolf Prize winner, wrote 'I believe that in mathematics, as everywhere else, you can say that something is correct or not only if you have understood this yourself. Since we do not have time to do everything, in mathematics I tend to believe that something is correct if I can understand some pieces of the proof or theory. If I do not understand anything, I try to refrain from making judgement.'[7] This attitude is

[7] personal communication, January 2018

shared by many mathematicians, and the vast majority of mathematicians does not say anything about anabelian geometry and IUT since they have not studied them.

Almost all negative sentiments about IUT on the internet originate from USA, the country where there are currently no active experts with (peer reviewed) research track record even in anabelian geometry. No French, British, Russian, Chinese or Japanese mathematicians are known to have ever publicly said or written anything negative about IUT. What makes the US mathematical climate different?

3.3. The study of IUT by two German mathematicians. When one does not apply appropriate efforts to study the area of a fundamentally new theory, one does not become an expert in it, whatever one's own different area of specialisation is and achievements in it. It is irresponsible to make one's general negative opinion about math work public when it is not based on its good knowledge. Of course, it is still possible to contribute useful questions/comments/remarks in relation to more conventional parts of the theory. In 2013–2017 no concrete mathematical remarks originated from mathematicians making negative public remarks about IUT. The only concrete remarks were made only in 2018 by Scholze and Stix. They were mathematically incorrect, most likely because of rushed efforts and small time spent on IUT, several times smaller than the efforts and time invested by many other people who have become experts on IUT. In comparison, diligent efforts by other learners to study IUT and ask interesting questions deserve more publicity.

Since 2014 or even earlier Scholze started to tell publicly at various workshops about 'faults' in IUT. I wrote to him a couple of times asking to do things properly: tell precisely what were the 'faults' with IUT he knew and to communicate with the author of IUT instead of publicly telling negative things about IUT. He declined to participate in our 2015 Oxford IUT workshop where one could learn many relevant things. Eventually, Scholze sent only one very vaguely and broadly stated question to Mochizuki in May 2015. The author of IUT responded to him with a long email, also kindly offering to conduct discussions via email to address any questions. Scholze responded he was too busy and stopped the email correspondence at that time. Part of this is stated on p.3 of the main Mochizuki's report[8] at his new page[9] where all files deserve attention. Scholze continued to publicly talk that something was wrong with IUT. I invited him to visit and discuss, to no avail. After a long time, he came with Stix to Kyoto in March 2018 just for 5 days. Apparently without any ready 'faults' to indicate and searching for some other 'errors' in IUT. Mochizuki writes about this further on the same p.3, 'On the other hand, the March 2018 discussions centered around quite different issues, such as (Ind1,2)'. Ind1,2,3 are three fundamental indeterminacies one needs to allow in order to have certain functoriality/multiradiality in IUT. By the end of their short visit Scholze–Stix came with their own strange version of IUT based on their fundamentally incorrect oversimplification of IUT by identifying all isomorphic rings with one another. In particular, the position of Scholze–Stix weirdly denies the use of anabelian geometry in IUT. This oversimplification is inappropriate for anabelian geometry and IUT, and the reaction of experts to this oversimplification can be read in sect.18 of the report[10]. This oversimplification strikes as incorrect even people far from number theory, e.g. math physicists and categorists.

Mochizuki's work in anabelian geometry has never been criticised. It is the fact that at least 80% of IUT is actually some advanced anabelian geometry. Without knowing it very well, one cannot progress with the study of IUT. It is not a problem to make a mistake, when one tries to understand a complex theory. However, talking about 'faults' in IUT for such a long time without having any valid evidence of the 'faults' is ungraceful, and so is the absence of visible efforts to answer to the comments of the author of IUT and to acknowledge one's mistakes.

[8] `http://www.kurims.kyoto-u.ac.jp/~motizuki/Rpt2018.pdf`

[9] `http://www.kurims.kyoto-u.ac.jp/~motizuki/IUTch-discussions-2018-03.html`

[10] see footnote 8

Initially, Scholze and Stix were keen to put their report about the meeting online. However, after reading Mochizuki's reports, see especially its sect. 17-18[11] and these comments[12], they completely changed their mind in July and stopped to be interested to post their own report. They eventually agreed to let the author of IUT to include their report on his pages. The second version of their report does not address most of comments of Mochizuki on their first report, thus showing their lack of interest to study IUT properly. The second version also includes new incorrect statements such as a blunder in classical height theory and a crucial misunderstanding of the famous Faltings proof.

3.4. Campaign of ignorant criticism of IUT. It originates from a very small group of researchers who know each other, as well as Scholze, well. None of them was a pundit in anabelian geometry or IUT and none has demonstrated any recently acquired expertise in these areas. None of them has revealed during 6 years any valid mathematical problem with IUT, but they keep telling negative incorrect things about IUT publicly.[13]

Just one example. Prior to our 2015 Oxford IUT workshop B. Conrad declined to prepare and give a talk on some rather easier topic. During the workshop he kept asking, often in inconsiderate way, irrelevant questions which slowed down the pace of the workshop and caused unnecessary upheavals. His questions demonstrated that, except its classical part, he came unprepared for the workshop despite the organisers recommendations[14] made prior to the workshop. Several speakers complained about his behaviour and one speaker warned him not to disturb his talk. After the workshop he rushed to post a text which revealed lack of fundamental understanding of some basic things, and it contained dozens of mistakes and incorrect comments.

What are the origins and reasons of very irresponsible and disrespectful towards the author of and experts in IUT public behaviour? Why did very few mathematicians decide to make public their opinions about a fundamental development in the subject area where they have empty research record and which they have not studied in any serious way? There is something rotten there. When grown up mathematicians publicly negatively talk about mathematics they do not know, unless they are very reckless, they have some concrete aims and reasons which have nothing to do with the theory they criticise. An example of organised disinformation is a recent article in a journal called quanta. That article consists almost entirely of opinions of non-specialists. Its author (not a professional mathematician) declined suggestions to make the article accurate and much less biased by increasing the ratio of experts opinions from current almost zero, by correcting numerous incorrect statements, and by including information about the level of expertise in the subject area of each cited mathematician.

4. Evaluating perspectives. A genuine consensus about any mathematical theory can only come from experts in its subject area. It cannot come from mass media publications or from the internet chats. As far as IUT is concerned, every mathematician can start to seriously study anabelian geometry and IUT, and his/her new math questions are welcome and answers will be given. Ignorant in the subject area of a new theory opinions of people who do not apply serious efforts to become experts in it have and should have zero value. To help the consensus about IUT among larger groups of mathematicians, non-specialists spreading ignorant disinformation should change their behaviour and attitude. To conduct a serious debate about mathematics of IUT with wider audience is hardly possible, since all of its participants should at least be familiar rather well with anabelian geometry and already this task requires some substantial time investment.

More surveys of IUT will help to educate the general audience. The number of experts in IUT will grow, there will be many new interesting developments around IUT. The situation in the mid- and long-term should become better, provided that many more become familiar with anabelian geometry, i.e. it becomes part of good graduate level education. At the same time, one can foresee, knowing the history of other fundamental pioneering developments, a certain ignorant opposition.

[11] see footnote 8

[12] http://www.kurims.kyoto-u.ac.jp/~motizuki/Cmt2018-05.pdf

[13] «Вот то-то мне и духу придает, что я, совсем без драки, могу попасть в большие забияки».

[14] https://www.maths.nottingham.ac.uk/plp/pmzibf/stiut1.pdf