ABOUT CERTAIN ASPECTS OF THE STUDY AND DISSEMINATION OF SHINICHI MOCHIZUKI'S IUT THEORY

IVAN FESENKO

This text aims to communicate in a compact form some of factual information related to the study of Shinichi Mochizuki's IUT theory[1] and its dissemination, as well as some related aspects including non-experts behaviour. Some of more general issues are discussed in two other papers[2] [3]. Without repeating the content of those papers, this text deals with some concrete issues, rather related to some mathematicians than to mathematics. The summary of what follows is this.

Number theory consists of many different areas and the distance from one area to the rest can be large[4]. The main prerequisite for IUT is the vast area of (arithmetic) anabelian geometry developed since 1990 in Japan, and the distance from it to most other areas of number theory is large. In 2012 there were few authorities in arithmetic anabelian geometry outside Japan. IUT is not an increase of mathematical knowledge in an area in which there are many specialists able to study it. It is a rare pioneering vast development with many new concepts and ideas, and with a great potential for future developments. To become a pundit in IUT, one has to invest an adequate large amount of time in a dedicated serious focused study of the theory starting with its prerequisites such as anabelian geometry. This cannot be done in the period of few weeks or months. To help mathematicians to study IUT, a very large amount of time and effort has been dedicated to the dissemination of IUT, via various workshops, including large international, via seminars, lectures and study groups. No valid math evidence of any serious fault in IUT, confirmed by professionals, has been found by anyone. Minor oversights have been found and corrected. To this day there remains no mathematically substantive reason whatsoever to doubt the validity of IUT. The number of researchers who have mastered IUT, by investing a large amount of time and effort, is steadily growing. Learners of IUT have sent a 4-digit number of questions and remarks to the author, all addressed. A two-digit number of surveys of IUT and a highly popular book on IUT, by mathematicians from several countries, individually present the theory in different ways. There are more mathematicians able to produce professional reports on the IUT papers than the number of such reports on previous very rare major math breakthrough at the time of their publication. In 2018–2019 two year long IUT seminars at RIMS for new learners were conducted. 2021 is a special RIMS year with 4 international workshops on anabelian geometry, combinatorial anabelian geometry and IUT.[5]

A number of researchers in the course of several years of their study of IUT have asked interesting or deep questions and contributed to new original developments. Some other researchers have tried to study IUT just for a short while, without attending various workshops on IUT, and failed. A tiny group of researchers, based not on their knowledge of anabelian geometry but on ignorance, were active in publicly making negative absurd

Date: last update April 2020.

[1] The IUT papers made public in August 2012 are available from section Inter-universal Teichmüller Theory of its author page `http://www.kurims.kyoto-u.ac.jp/~motizuki/papers-english.html`. See those pages for various information on seminars and workshops on IUT. See also this page `https://www.maths.nottingham.ac.uk/plp/pmzibf/guidestoiut.html`.

[2] I. Fesenko, Arithmetic deformation theory via arithmetic fundamental groups and nonarchimedean theta functions, notes on the work of Shinichi Mochizuki, Europ. J. Math. (2015) 1:405–440, available from `https://www.maths.nottingham.ac.uk/plp/pmzibf/notesoniut.pdf`

[3] I. Fesenko, Remarks on aspects of modern pioneering mathematical research, available from `https://www.maths.nottingham.ac.uk/plp/pmzibf/rpp.pdf`

[4] this also depends on the stage of development of the area

[5] see also an interview to the AMS available at `http://www.ams.org/news?news_id=3711`

remarks about IUT, always devoid of any valid math evidence of faults in the theory.[6] Few researchers, lacking any experience of working with central objects of anabelian geometry, chose to spread fake news and disinformation about IUT. Their reckless behaviour might have affected some people unable to distinguish an expert from a non-expert. Even worse, their action might have discouraged some potentially great mathematicians of the future to start and continue their long-term work on on key problems and theories.

1. On mathematical environment around IUT, briefly. Class field theory, the heart of algebraic number theory, has several important generalisations. They include the Langlands correspondences, anabelian geometry and higher class field theory. By various reasons the first generalisation[7] has attracted many times more researchers than the second and the third, but all of these generalisations of class field theory are fundamentally important. Most of the central problems in the second and third generalisations of class field theory have been settled[8]. One can imagine another universe where higher class field theory and anabelian geometry attract many more researchers than in this universe and where general class field theory concepts are well understood by all number theorists.

The main prerequisite for IUT theory is arithmetic anabelian geometry, including Mochizuki's famous proofs of the Grothendieck conjecture and his absolute and mono-anabelian geometry. Arithmetic anabelian geometry was started in works of Neukirch–Ikeda–Uchida–Iwasawa for small fields (such as number fields or their completions) in characteristic zero, and from a different motivation for hyperbolic curves over number fields it was proposed by Grothendieck. The main leading country in arithmetic anabelian geometry is Japan, and the first three contributors to anabelian geometry for hyperbolic curves were H. Nakamura, A. Tamagawa and Sh. Mochizuki.[9] Below 'anabelian geometry' will mean 'arithmetic anabelian geometry'. In the last thirty year a vast body of fundamentally important results in anabelian geometry were established. These developments were essentially left unnoticed in many countries and outside a small group of experts.

The IUT theory uses some key theorems in anabelian geometry, as well as its later developments such as absolute anabelian geometry and mono-anabelian geometry. The total volume of relevant papers in anabelian geometry used in one or another extent in IUT is huge. One starting observation for arithmetic deformation theory, i.e. IUT, is that unlike the usual algebraic geometry in which working with schemes locally corresponds to working with commutative rings, working with certain anabelian objects corresponds to working with large nonabelian topological groups, thus using one operation instead of two, with new options to perform kinds of arithmetic deformation, not available in the standard arithmetic geometry. There is an associated difficulty to measure the deviation of certain diagrams of groups and maps between groups from being commutative, which IUT solves, thus eventually providing bounds on certain deformations.

2. The study of IUT. Links to various study materials about IUT are available from pages of the author of IUT[10]. The total amount of time dedicated to the verification process of IUT by mathematicians is several decades, and it looks to be the largest time ever spent in the history of mathematics on the verification of a mathematical work prior to its publication. Several international conferences were organised in 2014-2016. Numerous intensive seminars have been held in Japan (2012-2019), UK (2015-2018), China (2015-2016), they

[6] compare with 'we are also fighting on a second front that we did not anticipate, fighting a battle against misinformation and disinformation in a hyper-partisan environment ... the world has changed in profound ways since even 2010. Social media, hyper-partisanship, the broad populist distrust of experts, plummeting standards of factfulness', `https://twitter.com/CT_Bergstrom/status/1243252341756669953`

[7] even though it is still lacking a version parallel to general class field theory, see the next footnote

[8] for more details and related math issues see `https://www.maths.nottingham.ac.uk/plp/pmzibf/232.pdf`

[9] In the 1990s, a series of results about anabelian properties of Galois groups of global and higher global fields, i.e. birational anabelian geometry, were obtained by F. Pop. Since the early 1990s, F. Bogomolov suggested and developed, later in collaboration with Yu. Tschinkel, his birational anabelian geometry for varieties of dimension > 1 over algebraically closed fields, this theory is quite different from arithmetic anabelian geometry in many respects.

[10] `http://www.kurims.kyoto-u.ac.jp/~motizuki/top-english.html`

involved nationals of many countries. In addition to the referees' comments, active learners of IUT sent a 4-digit number of comments, questions, remarks, all had been carefully taken into account by the author. Several learners of IUT shared their understanding of it, by having written texts and surveys. There are already more surveys of IUT than of any previous rare breakthrough work at the time of its publication.[11]

Small numbers of experts in anabelian geometry, a relatively poor digestion of Grothendieck's 50 years old math heritage by number theorists, a large distance from anabelian geometry and IUT to currently fasionable directions, a large number of new concepts in IUT and its large volume, as well the general situation of substantially weakened research in number theory in many countries have affected the study IUT and its perception.

Some people applied serious efforts to study IUT for some time, without attending IUT workshops, but stopped. This is normal in relation to the study of complex theories. Whatever are one's previous contributions to other areas of number theory, those do not make one an authority in anabelian geometry and IUT.

Recommendations to mathematicians interested to study IUT. Pathways to study IUT are available from many sources including www-links in footnotes of this text. If you find a piece of IUT looking to you as an error and you cannot resolve it, document your evidence and contact the author or its learners to discuss. Some may be waiting for a ready-to-digest version of the theory but it will take time for it to be produced.[12]

3. On negative aspects of reaction to IUT.

3.1. On reaction to IUT from some mathematicians. Many mathematicians are interested to know more about IUT. As always, they can ask experts. Talking exclusively with non-experts, who have very weird ideas about IUT, can only produce weird outcomes. And it is possible to contribute useful questions, comments, remarks, e.g. in relation to more conventional parts of the theory, e.g. such as those that came in 2012.

As follows from various discussions, mathematicians who can understand *conventional* things quickly, who can master a domain that *has already been well established* may have not so good chances to progress in the study of IUT.[13] Several expected candidates to study the theory chose to be as unambitious as it can get by doing essentially nothing about its study at all, i.e. for more than seven years. Some preferred to adopt the convenient stance of sceptical attitude not based on professional knowledge of the subject area.

One can occasionally hear a naive request to provide more details and explanations for the IUT papers, with an associated psychologically comfortable attitude to wait for this to happen. This demonstrates the sheer lack of basic knowledge of the situation: the level of presentation of IUT was already very detailed in 2012 and in the last 7.5 years the volume of the IUT papers had increased by some 100 pages more, reaching almost 600 pages, and there are already many surveys of IUT and even a book for the general audience.

Mochizuki's work includes fundamental contributions in numerous directions: Hodge–Arakelov theory, anabelian geometry, mono-anabelian geometry, combinatorial anabelian geometry, Grothendieck–Teichmüller group, p-adic Teichmüller theory, inter-universal Teichmüller theory. Except for the last direction, none of his work has ever been criticised because it was read and appreciated by experts in the subject area. A. Beilinson wrote 'I believe that in mathematics, as everywhere else, you can say that something is correct or not only if you have understood this yourself. Since we do not have time to do everything, in mathematics I tend to believe that something is correct if I can understand some pieces of the proof or theory. If I do not understand anything, I try to refrain from making judgement.'[14] This attitude is shared by most mathematicians. By reasons known to themselves, few mathematicians chose to talk in a benighted way about IUT and its study, while being fully aware they simply do not have any authority in the subject area. They made public their ignorant negative opinions about a fundamental development in the subject area where they have empty research record, with no evidence of their serious study of it, and without providing any math evidence of errors in the theory. In 2012

[11] For their incomplete list see e.g. `http://www.maths.nottingham.ac.uk/plp/pmzibf/guidestoiut.html`.

[12] Galois theory was 'notoriously difficult for his contemporaries to understand, especially to the level where they could expand on it', and its digestion took decades; its best presentation by E. Artin appeared many decades later.

[13] compare with `https://www.bbc.co.uk/news/world-europe-50856999`

[14] personal communication, January 2018

there was only US researcher[15] knowledgeable of certain parts of arithmetic anabelian geometry, while most of incorrect negative comments originate from a tiny group of mathematicians in that country.

Some chose to spread a malicious distortion of the math truth or false rumours[16]. One of them is talking about some kind of controversy about the status of IUT — however, to have a controversy about a mathematical work there should be genuine experts on both sides of the argument able to provide valid math arguments, but this is plainly not the case for IUT. This also explains why not the trial of a serious math peer review but the choice of shallow posting is the only venue for their non-expert public chats about IUT. A related disinformation is that there are allegedly two professional math sides in their take on IUT. This is incorrect: there is only one side, the side of experts in IUT, which includes many those who have worked for years to learn the subject area and the theory. They, together with the referees and the group of editors processing the IUT papers, have all concluded that the IUT papers have no mathematical flaws. Part of this process was a truly unprecedented event with the author of IUT kept investing a lot of time in answering more than 1000 of questions for more than 7 years.[17]

3.2. Some articles about IUT in mass media. IUT has attracted a high level of interest from mass media. There are rare reasonably good written articles about IUT and its author, in particular in Japanese media. At the same time, there are articles presenting very wrong pictures by journalists lazy to do their work properly.

Most pundits in IUT decline to answer journalists questions, so then journalists contact mathematicians who are not experts in anabelian geometry or even laypersons with zero publication record in number theory. Some of the interviewed mathematicians are good in their own areas, but that does not make them authorities in areas they simply do not know: experience in areas such as classical Diophantine geometry, algebraic geometry, modularity, Galois representations or aspects of p-adic geometry does not enable one with the required intuition and knowledge of anabelian geometry and IUT. Interviewed mathematicians are well aware of that.

One of easiest ways for journalists to write their articles is to present opposite points of view but in the case of IUT the journalists often fail to appreciate that they mix experts opinions (all of which are positive) with ignorant opinions of non-specialists who are not kept in the loop in relation to the study of IUT.

Recommendation to serious journalists. Before interviewing a mathematician about IUT, first ask several simple questions such as their knowledge of and expertise in anabelian geometry, delivered talks on anabelian geometry at international conferences, the number of hours spent on the study of IUT and whether they asked questions about IUT to the author of IUT or experts in IUT.

3.3. One among many attempts to study IUT. In comparison to a lot of good questions on IUT by its serious learners, a number of which deserves publicity and appreciation, one rushed unsuccessful attempt by two German mathematician became more publisised. In 2013–2017 not a single concrete mathematical remark indicating any essential issue in IUT was produced. P. Scholze, with no publications or expertise in anabelian geometry, kept talking publicly about faults in IUT since 2014 without ever providing any math evidence.[18] By and by, after a lot of pressure, he visited RIMS, together with J. Stix, in March 2018, just for few days. They were asked to produce a report about their study of IUT so that any mathematician can read it. Their first report which includes a very short presentation of their understanding of IUT is a caricature version of IUT, based on a gross oversimplification of IUT in which they identify all isomorphic rings and 'forget' about the fundamental role of automorphism groups in anabelian geometry. One cannot be surprised that they conclude that their

[15] who chose not to study IUT

[16] See e.g. the report about the Oxford IUT workshop `https://www.maths.nottingham.ac.uk/plp/pmzibf/files/iut-i-rep.html`

[17] compare this intensive study and verification with the next section material

[18] The author of this text wrote to Scholze several times asking to behave professionally and in particular to tell precisely what were the faults in IUT he knew about and discuss with experts, but no response had come. Eventually, Scholze sent just one most loosely stated question to Mochizuki in May 2015. The author of IUT responded to him with a long email that also offered to conduct discussions via email to address any questions, but Scholze declined to communicate further. Part of this is stated on p.3 of the main Mochizuki's report, see footnote 20.

version is incorrect, but this has nothing to do with IUT. Moreover, the report essentially denies the use of anabelian geometry and infinitely many theatres in IUT[19]. One does not need to progress in the study of anabelian geometry and IUT to an advanced level to easily observe how absurd is their caricature of IUT. The reaction of experts to their report can be read in sect.18 of a report[20] of the author of IUT. The German mathematicians intended to make their report available online, however, after reading the comprehensive report[21] of the author of IUT on their report, see especially its sect. 17-18[22] and these comments[23], they changed their mind to the extent of abandoning any plans to post their own report. In his comprehensive report on their report the author of IUT formulated few questions to the German mathematicians which may have helped them to appreciate how erroneous was their take on IUT.[24] However, the second version of their report failed to address those few questions. Moreover, it included new incorrect statements demonstrating inadequate knowledge of more classical areas such as a blunder in height theory and a fundamental misunderstanding of one of the Faltings work, and those reckless mistakes can be easily seen by researchers not familiar with anabelian geometry. Scholze unilaterally withdrew from any further correspondence or study of IUT. This rushed study of IUT, accompanied with the inability to answer few questions asked them by the author of IUT in his first report[25], is rejected by all experts in IUT; it simply can not pass any careful peer review process. The author of IUT had to include their reports on his pages, so that any researcher can directly check their numerous flaws.[26] That 'study' of IUT by the two mathematicians[27] stands in shark contrast with the diligent study of it by a two-digit number of other researchers who, as most serious mathematicians, do not use blogs to express their knowledge and opinions.

4. Developments. There will be several new math developments related to IUT, in different directions.

Learners of IUT can attend four international workshops on anabelian geometry and IUT are organised during a special RIMS Project Research year on Expanding Horizons of Inter-universal Teichmüller Theory in 2021[28], supported by the new Center for Research in Next-Generation Geometry.

A book[29] by F. Kato, published in April 2019, presents various features of IUT to the wider audience. This book was in the list of top twenty bestselling books in all subject areas on amazon in Japan, and it was awarded the Yaesu prize[30].

On April 3 2020 at the press-conference of Kyoto University, ran by M. Kashiwara and A. Tamagawa, it was announced that the IUT papers are accepted for publications and will soon be published.[31]

---

[19] For a popular presentation to high school students of the importance to use infinitely many theatres in IUT, one can watch F. Kato's talk `https://www.youtube.com/watch?v=fNS7N04DLAQ&vl=en`

[20] the main report `http://www.kurims.kyoto-u.ac.jp/~motizuki/Rpt2018.pdf` at the page `http://www.kurims.kyoto-u.ac.jp/~motizuki/IUTch-discussions-2018-03.html`.

[21] referred to in footnote 20

[22] see references in footnote 20

[23] `http://www.kurims.kyoto-u.ac.jp/~motizuki/Cmt2018-05.pdf`

[24] see also Remarks 3.11.1 and 3.12.2 of IUT-III

[25] as compared to over a thousand of questions answered by the author of IUT

[26] Putting their report on the page of the author of IUT does not imply in any way its validity, of course.

[27] Compare with the content of published in 1931 book 'Hundert Autoren gegen Einstein' characterised as 'a reaction of an inadequately educated academic citizenship, which didn't know what to do with relativity'and as an 'accumulation of naive errors', at least its authors cared to publish their opinions.

[28] `http://www.kurims.kyoto-u.ac.jp/kyoten/en/index.html`, `http://www.kurims.kyoto-u.ac.jp/~motizuki/project-2020-english.html`

[29] `https://twitter.com/FumiharuKato`

[30] `https://twitter.com/yaesu_paseo/status/1190084381529886721?ref_src=twsrc%5Etfw`

[31] `https://www.sankei.com/life/news/200403/lif2004030104-n1.html`, `https://www.sankei.com/life/news/200403/lif2004030075-n1.html`, `https://www.sankei.com/life/news/200403/lif2004030058-n1.html`, `http://www.asahi.com/ajw/articles/13271575`