THE STATISTICAL ANALYSIS OF SHAPE DATA

by

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ABSTRACT

In this thesis we consider some topics connected with a statistical shape analysis of point set data. We first give the exact shape distribution for a finite number of points which are independent isotropic bivariate normally distributed in a plane. Various properties of the distribution are investigated, including an asymptotic large variation distribution and a normal approximation for small variations. Connections with previous work are made, and moments, marginal distributions and invariances are also considered. The shape density for triangles is examined in particular detail.

The exact shape distribution is then used in a likelihood based inference procedure, which can be implemented on a computer. Various estimates are compared in a simulation study and the approximate inverse Fisher information matrix is also given. Likelihood ratio testing for shape change is examined and in particular we describe testing for a uniform shear - the simplest possible shape change. Inference is illustrated with a mouse vertebrae study from anatomy.

As a natural extension, the most general multinormal model in a plane is proposed. The exact shape probability density function under this model is given in a closed form. Although the density is quite complicated, it simplifies considerably in certain cases. Various properties are considered, including a useful normal approximation. Likelihood based inference with this general model is not straightforward, although we shall consider a simple anisotropic model for the mouse vertebrae data.

Some practical issues are discussed and an algorithm for semi-automatic landmark location is proposed. Finally, as a suitable summary for describing a shape change we consider an alternative to the biorthogonal grids for visualizing size and shape change.

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ABBREVIATIONS

MLE	-	Maximum Likelihood Estimate
MSE	-	Mean Square Error
NAG	-	Numerical Algorithms Group, Oxford.
pdf	-	probability density function
T1	-	First Thoracic Vertebra
T2	-	Second Thoracic Vertebra