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Regularity conditions for Banach function algebras

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Banach function algebras are complete normed algebras of bounded, continuous, complex-valued functions defined on topological spaces. There are very many different examples with a huge variety of properties. Two contrasting examples are the algebra of all continuous complex-valued functions on the closed unit disc, and the subalgebra of this algebra consisting of those functions which are continuous on the closed disc and analytic on the interior of the disc. In the second of these algebras, any function which is zero throughout some non-empty open set must be constantly zero. This is very much not the case in the bigger algebra: indeed Urysohns lemma shows that for any two disjoint closed subsets of the closed disc, there is a continuous, complex-valued function defined on the disc which is constantly 0 on one closed set and constantly 1 on the other (algebras of this type are called regular algebras).

Most Banach function algebras have some features in common with one or the other of these two algebras. In particular, Banach function algebras may satisfy one or more "regularity conditions", by which we mean conditions related in some way to the regularity of the algebra. Such conditions have important applications in several areas of functional analysis, including automatic continuity theory and the theory of Wedderburn decompositions. There is also a close connection between regularity and the theory of decomposable operators on Banach spaces.

This mini-course will begin with revision of the basic definitions and Gelfand theory of commutative Banach algebras, before moving on to discuss the conditions under consideration, the relationships between them, and their applications. There will also be numerous illustrative examples of Banach function algebras and uniform algebras, and a discussion of the latest developments in this area.

Many of the examples and conditions may be found in Chapter 4 of the book 'Banach Algebras and Automatic Continuity' by H. Garth Dales, London Mathematical Society Monographs, New Series, Volume 24, The Clarendon Press, Oxford, 2000. Most of Joel Feinstein's e-prints are available from http://www.maths.nott.ac.uk/personal/jff/Papers/