NOTES AND COMMENTS ON

HOW DO WE DO PROOFS?
(PART I) 2008-9 EDITION

PLEASE COLLECT HANDOUT FROM THE FRONT

These notes should be read in association with the handout / pdf presentation.

n an odd integer:
prove $n^4 - 1$ is divisible by 8.

"n is odd" means $\exists\, k \in \mathbb{Z}$ s.t. $n = 2k+1$. (Or any equivalent, correct definition.)

"m is divisible by 8" means $\exists\, \ell \in \mathbb{Z}$ s.t. $m = 8\ell$.

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$\left(\text{Generally } (a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k . \right)$$

<u>Proof of result.</u>

Let n be an odd integer.
Then there is a $k \in \mathbb{Z}$ with
$n = 2k+1$.
We have

$$n^4 - 1 = (2k+1)^4 - 1$$

$$= \left((2k)^4 + 4(2k)^3 + 6(2k)^2 + 4(2k) + 1\right) - 1$$

$$= 16k^4 + 32k^3 + 24k^2 + 8k$$

$$= 8(2k^4 + 4k^3 + 3k^2 + k).$$

So $n^4 - 1$ is divisible by 8. $\square$

Question 2. Prove that, for every pair of odd integers m and n, $m+n$ is even.

Reformulation

The fact we are trying to prove can be reformulated as follows:

"Let m and n be odd integers. Then $m+n$ is even."

Make sure you can write down definitions of "odd" and "even".

Proof. Let m and n be odd integers. Then there are k and ℓ in $\mathbb{Z}$ with

$$m = 2k+1 \quad \text{and} \quad n = 2\ell+1.$$

Then

$$\begin{aligned} m+n &= (2k+1)+(2\ell+1) \\ &= 2k+2\ell+2 \\ &= 2(k+\ell+1). \end{aligned}$$

Thus $m+n$ is even. $\square$