How do we do proofs? (Part II)

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Definition

A sequence (x_k) of real numbers is said to be **strictly increasing** if we have

$$x_1 < x_2 < x_3 < \cdots .$$

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Question 1 (formal justification not required)

Let (n_k) be a strictly increasing sequence of natural numbers.

- What is the smallest possible value that n_2 could have? What about n_{200}?
- For a given positive integer k, the minimum possible value that n_k could have clearly depends on k. In terms of k, what is this minimum possible value?
  Your answer should be a function of k.
- If you **did** have to justify your answers formally here, how would you do so?

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These questions will build on our discussion above, and also on your previous knowledge of results concerning the convergence and divergence for series.

Note again that, for us, 0 will **not** count as a natural number: for us $0 \notin \mathbb{N}$, and so

$$\mathbb{N} = \{1, 2, 3, \ldots\}\,.$$

Question 2

Prove that, for every strictly increasing sequence of natural numbers (n_k), the series $\sum_{k=1}^{\infty} \frac{1}{n_k^2}$ is convergent.

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Question 3

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Meta-question 2B

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Meta-question 2C

What sort of things will you need to assume, state or use during the proof?

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Meta-question 3C

Given that you are expected to justify your answer fully, what expectations are implicit in Question 3, i.e., what do you have to do to give a satisfactory answer to Question 3?