G12MAN Mathematical Analysis
How do we do proofs?

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2006-2007

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6. How many different kinds of proof are there?

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If we wish to work with sequences starting from a different first term, we will specify this. For example, a sequence of the form $x_0, x_1, x_2, \ldots$ can be denoted by $(x_k)_{k=0}^{\infty}$.

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Question 3

Let (n_k) be a strictly increasing sequence of natural numbers.

- What is the smallest possible value that n_2 could have?
- For a given positive integer k, what is the smallest possible value that n_k could have?
- What happens if you use the 'other' definition of $\mathbb{N}$?

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I accept that the answers to Question 3 are **clear**, and can (and should) be stated without proof in this module (by saying **'clearly, . . . '**).

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Before attempting the next two questions, you should attempt the meta-questions (i.e. questions about questions) which follow them.

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These questions are not particularly hard in themselves, though they require some understanding of convergence and divergence for series. It is the illustrative meta-questions which follow which are more important here.

Two questions about series

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Prove that, for every strictly increasing sequence of natural numbers (n_k), the series $\sum_{k=1}^{\infty} \frac{1}{n_k^2}$ is convergent.

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Meta-question 5B

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Meta-question 5C

What are you allowed to assume, state or use during the proof?

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Do you have a guess as to what the answer might be?

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Meta-question 6B

Do you have a guess as to what the answer might be?

Meta-question 6C

Given that you are expected to justify your answer fully, what expectations are implicit in Question 6, i.e., what do you have to do to give a satisfactory answer to Question 6?