Convergence and divergence for some standard series

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The following is a list of some useful, standard series from G11ACF, with which you should be familiar. This list is not necessarily comprehensive.

1 Geometric series

Let $x \in \mathbb{R}$. Then the series

$$\sum_{k=1}^{\infty} x^k$$

is convergent if $|x| < 1$, and is divergent otherwise. In particular,

$$\sum_{k=1}^{\infty} \frac{1}{2^k}$$

is convergent, while

$$\sum_{k=1}^{\infty} (-1)^k$$

is divergent.

2 Powers of k

Let $p \in \mathbb{R}$. Then the series

$$\sum_{k=1}^{\infty} k^{-p}$$

converges if $p > 1$, and diverges otherwise. In particular,

$$\sum_{k=1}^{\infty} \frac{1}{k}$$

diverges, while

$$\sum_{k=1}^{\infty} \frac{1}{k^2}$$

converges.