Why do we do proofs?

Joel Feinstein

School of Mathematical Sciences
University of Nottingham

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5. Why are definitions of concepts important?
6. Do we need to memorize lots of proofs?

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Prove that, whenever x, y, z and n are positive integers with $n > 2$, then

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That's rather a tricky one. It took over 350 years to find a proof!

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Problem 3 (Serious answers please!)

Given that none of the numbers involved are zero, is it possible that both hospitals have got their calculations right?
If so, which hospital would you rather be treated by?

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