

# Why do we do proofs?

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- 5 Why are definitions of concepts important?
- 6 Do we need to memorize lots of proofs?

## Problem 1

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That's rather a tricky one. It took over 350 years to find a proof!

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### **Problem 3 (Serious answers please!)**

Given that none of the numbers involved are zero, is it possible that both hospitals have got their calculations right?

If so, which hospital would you rather be treated by?

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#### Problem 6

Which of the following numbers are prime? (a) 2 (b) 1 (c)  $-3$ .