Why do we do proofs?

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4. Why should we prove statements which appear to be intuitively obvious?
5. Why are definitions of concepts important?
6. Do we need to memorize lots of proofs?

Some problems

Problem 1

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That's rather a tricky one. It took over 350 years to find a proof!

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Problem 3

Given that none of the numbers involved are zero, is this possible? If so, which hospital would you rather be treated by?

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Problem 4

Is every square a rectangle?