Gravity as a Gauge Theory

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Plan of the talk:

- Explain how a gauge field can be taken as the dynamical variable in gravity (instead of metric)

  Eddington's formulation of GR

  Gravity as an SU(2) gauge theory

- Explain how the gauge-theoretic formulation leads to a number of simplifications

  Absence of the conformal mode problem

  Simplifications in the perturbation theory

- Possible applications to quantum gravity and unification

Eddington's reformulation of GR

No need to have a metric to know how particles move

tangent vector to a trajectory satisfies

$$v^\nu \nabla_\nu v^\mu = 0$$

need affine connection $\Gamma^\rho_{\mu\nu}$ (symmetric)

$$\Gamma \rightarrow Riem \rightarrow Ricci$$

Defining the metric

$$\boxed{g_{\mu\nu} := \frac{1}{\Lambda} R_{\mu\nu}}$$

need $\Lambda \neq 0$

Action principle for $\Gamma^\rho_{\mu\nu}$

$$S_{\mathrm{E}}[\Gamma] = \frac{\Lambda}{8\pi G \Lambda} \int \sqrt{\det g_{\mu\nu}(\Gamma)}$$

just the volume!

Field equations

$$\nabla_\mu R^{\mu\nu} = 0 \quad \Rightarrow$$

connection is metric-compatible

second-order PDE's for the affine connection

Eddington's functional from the first order formulation

$$S[g,\Gamma] = \frac{1}{16\pi G}\int \left(g^{\mu\nu}R_{\mu\nu}(\Gamma) - 2\Lambda\right)\sqrt{\det g}$$

eliminate Γ

eliminate g

algebraic equations to solve in both cases

not bounded from either below or above

$S_{\mathrm{EH}}[g]$

$S_{\mathrm{E}}[\Gamma]$

non-negative

different slices of the same first order functional

Shortcomings of Eddington's approach

- Einstein uses 10 components of the metric to describe 2 propagating DOF of the graviton

  Eddington uses 40 components of the affine connection for the same purpose

- Not easy to couple matter

  Possible by starting with the first order action, and integrating out the metric

- Requires $\Lambda \neq 0$

  Used to be the main argument against this formulation. Not anymore

Interesting features

- $S_{\mathrm{E}}[\Gamma]$ is bounded from below

- Only on-shell equivalent - possibly different quantum theory

  Still power-counting non-renormalizable

  Still one-loop renormalizable

  Nobody knows what happens at two loops

- Attempts were made to unify gravity with electromagnetism using this formulation

  Einstein, Schroedinger

$$\Gamma^{\rho}_{\mu\nu} \neq \Gamma^{\rho}_{(\mu\nu)}$$

This failed - good, there is more than just electromagnetism and gravity

This talk:

Gravity with $\Lambda \neq 0$ as a dynamical theory of an SU(2) connection

Obtained from Plebanski first order formulation
by integrating out the metric-like field

Why study this?

In Eddington's case 40 is much worse than 10

In our case the connection field A_μ^i $\quad i = 1, 2, 3$

4 x 3=12 components per point

Connections/gauge 12-3=9 per point

This description of gravity is more economical than
the metric-based one

A functional in the space of conformal metrics:

analogous to the one appearing in the Yamabe problem

Can integrate over metrics in two steps:

1) Conformal factor $g = e^{2\phi} g_0$

2) Conformal class $[g_0]$

After the first step get a functional that depends only on the conformal class - Yamabe functional $S[[g_0]]$

The "pure connection" functional is analogous but different

Pure connection formulation of GR

Importance of the isomorphism $\mathrm{SO}(1,3) \sim \mathrm{SL}(2,\mathbb{C})$

is universally appreciated

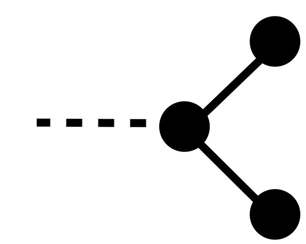
The pure connection formulation owes its existence to the following isomorphism

$$\boxed{\mathrm{SO}(6,\mathbb{C}) \sim \mathrm{SL}(4,\mathbb{C})}$$

Very important for twistor theory

Dynkin diagrams

$\mathfrak{sl}(n+1)$



$\mathfrak{so}(2n)$

Proof: Consider the 6-dimensional space Λ^2 of 2-forms in $\mathbb{R}$

The wedge product makes Λ^2 into a metric space

$$\Lambda^2 \ni U, V \to (U, V) = U \wedge V / d^4x \in \mathbb{R}$$

metric of signature (3,3) if over $\mathbb{R}$

$\mathrm{SL}(4, \mathbb{R})$ acts on Λ^2 $\qquad G_\mu^\nu \in \mathrm{SL}(4, \mathbb{R})$

$${}^G U_{\mu\nu} = G_\mu^\alpha G_\nu^\beta U_{\alpha\beta}$$

the wedge product metric is preserved

$$\Rightarrow \quad \mathrm{SL}(4, \mathbb{R}) \sim \mathrm{SO}(3, 3)$$

The isomorphism implies

$\mathrm{SL}(4)/\mathrm{SO}(4)$		$\mathrm{SO}(3,3)/\mathrm{SO}(3)\times\mathrm{SO}(3)$
	$\Leftrightarrow$	
conformal metrics on M		Grassmanian of 3-planes in Λ^2

$\boxed{\text{Conformal metrics can be encoded into the knowledge of which 2-forms are self-dual}}$

<u>Explicitly:</u> a triple of linearly independent 2-forms $B^i_{\mu\nu}$

$$\Rightarrow \quad g_{\mu\nu} \sim \tilde{\epsilon}^{\alpha\beta\gamma\delta}\epsilon^{ijk}B^i_{\mu\alpha}B^j_{\nu\beta}B^k_{\gamma\delta}$$

Urbantke formula

2-forms $B^i_{\mu\nu}$ are self-dual with respect to this metric

Formulation of the theory:

Let A^i be an SU(2) connection

$\left(\begin{array}{c}\text{SL(2,C) connection for}\\ \text{Lorentzian signature}\end{array}\right)$

$$F^i = dA^i + (1/2)[A,A]^i$$

$$F^i \wedge (F^j)^* = 0$$

reality conditions

declare F^i to be self-dual 2-forms $\Rightarrow$ conformal metric

To complete the definition of the metric need to specify the volume form

$$(\mathrm{vol}) := \frac{1}{2\Lambda^2}\left(\mathrm{Tr}\sqrt{F \wedge F}\right)^2$$

$$\boxed{S[A] = \frac{\Lambda}{8\pi G}\int (\mathrm{vol})}$$

as in Eddington's case, the action is just the volume

Functions of the curvature

Let f be a function on $\mathfrak{g} \otimes_S \mathfrak{g}$ satisfying

$\mathfrak{g}$ - Lie algebra of G

$$f : X \to \mathbb{R}(\mathbb{C})$$

$$X \in \mathfrak{g} \otimes_S \mathfrak{g}$$

defining function

1) $$f(\alpha X) = \alpha f(X)$$ homogeneous degree 1

2) $$f(gXg^T) = f(X), \ \ \forall g \in G$$ gauge-invariant

Then $f(F \wedge F)$ is a well-defined 4-form (gauge-invariant)

In practice:

define $\tilde{X}^{ij} := \frac{1}{4}\tilde{\epsilon}^{\mu\nu\rho\sigma} F^i_{\mu\nu} F^j_{\rho\sigma}$ so that

$$F^i \wedge F^j = \tilde{X}^{ij} d^4x$$

then $f(F^i \wedge F^j) = f(\tilde{X}^{ij}) d^4x$

Field equations:

(*) $$d_A \left(\mathrm{Tr}\sqrt{X} (X^{-1/2})^{ij} F^j \right) = 0$$

second-order PDE's for the connection

Theorem:

$\Lambda \neq 0$ KK PRL106:251103,2011

results on zero scalar curvature in early 90's Capovilla, Dell, Jacobson

For connections A^i satisfying (*) the metric $g(A)$ is Einstein with non-zero scalar curvature Λ

In the opposite direction, the self-dual part of the Levi-Civita connection for an Einstein metric satisfies (*)

Caveat: only metrics with $\Lambda/3 + W^+$ invertible almost everywhere covered

examples not covered
$S^2 \times S^2$
Kahler metrics

Perturbation theory (around de Sitter space)

a^i_μ infinitesimal SU(2) connection

Spinorial description: $\mu \to AA' \quad i \to (AB)$

$i = 1, 2, 3$

μ spacetime index

$$a^i_\mu \to a_{AA'}{}^{BC} \quad \in S_+ \otimes S_- \otimes S_+^2 = S_+^3 \otimes S_- \oplus S_+ \otimes S_-$$

$$\boxed{\mathcal{L}^{(2)} \sim \left(\partial^{(A}_{A'} a^{BCD)A'}\right)^2}$$

$a^{(ABC)}_{A'}$

$a_{A'E}{}^E{}_A$

pure gauge (diffeomorphisms) part

explicitly non-negative (Euclidean signature) functional

only depends on the $S_+^3 \otimes S_-$ part of $a_{AA'}{}^{BC}$

$\dim(S_+^3 \otimes S_-) = 8$ (per point)

Count of propagating DOF:

$8 - 3 - 3 \to 2$ propagating DOF

SU(2) gauge rotations

after gauge-fixing only 8 connection components propagate (an irrep of Lorentz!)

Interactions:

expansion around de Sitter $M^2 = \Lambda/3$

significantly more complicated expression in the metric case

<u>complete off-shell</u> cubic vertex

zero on-shell

$$\mathcal{L}^{(3)} = \frac{2}{MM_p}(\partial a)^{ABCD}(\partial a)^{M'N'}{}_{AB}(\partial a)_{M'N'CD} - \frac{1}{4MM_p}(\partial a)^{ABCD}(\partial a)_{AB}(\partial a)_{CD} + \frac{4M}{M_p}(\partial a)^{M'N'AB}(aa)_{M'N'AB}.$$

the only part that is relevant for MHV amplitudes

where the spinor contraction notations are

$$(\partial a)^{ABCD} = \partial^{(A}{}_{M'}a^{B)CDM'},$$

$$(\partial a)^{M'N'AB} = \partial^{C(M'}a_C{}^{ABN')}$$

$$(aa)^{M'N'CD} = a^{CD(AM'}a_{CD}{}^{B)N'}$$

Comparison with Yang-Mills:

can rewrite the YM Lagrangian as

gauge indices suppressed

$$\mathcal{L}_{\rm YM} = -\frac{1}{4g^2}(F^+_{\mu\nu})^2$$

F^+- self-dual part of the curvature

Spinorial description: $\mu \to AA'$

$$A_{AA'} \in S_+ \otimes S_-$$ spin 1

quadratic order (not gauge-fixed)

$$\mathcal{L}^2_{\rm YM} \sim (\partial^{(A}_{A'} A^{B)A'})^2$$

cubic order

$$\mathcal{L}^3_{\rm YM} \sim \left(\partial^{(A}_{A'} A^{B)A'}\right) A^{M'}{}_A A_{M'B}$$

our linearized graviton Lagrangian and the cubic vertex is just the generalization to the case

$$A_{ABCA'} \in S^3_+ \otimes S_-$$ spin 2

Einstein gravity perturbatively: Nasty mess...

Expansion around an arbitrary background $g_{\mu\nu}$

quadratic order (together with the gauge-fixing term)

$$L_{g.f.} = -\sqrt{-g}\left(h^{\mu\nu}{}_{;\nu} - \frac{1}{2}h_{\nu}{}^{\nu;\mu}\right)\left(h^{\rho}{}_{\mu;\rho} - \frac{1}{2}h^{\rho}{}_{\rho;\mu}\right)$$

$$L_2 = \sqrt{-g}\left\{-\frac{1}{2}h^{\alpha\beta}{}_{;\gamma}h_{\alpha\beta}{}^{;\gamma} + \frac{1}{4}h^{\alpha}{}_{\alpha;\gamma}h_{\beta}{}^{\beta;\gamma} + h_{\alpha\beta}h_{\gamma\delta}R^{\alpha\gamma\beta\delta} - h_{\alpha\beta}h^{\beta}{}_{\gamma}R^{\delta\alpha\gamma}{}_{\delta}\right.$$
$$\left. + h^{\alpha}{}_{\alpha}h_{\beta\gamma}R^{\beta\gamma} - \frac{1}{2}h_{\alpha\beta}h^{\alpha\beta}R + \frac{1}{4}h^{\alpha}{}_{\alpha}h^{\beta}{}_{\beta}R\right\}.$$

from Goroff-Sagnotti "2-loop" paper

cubic order

$$L_3 = \sqrt{-g}\left\{-\frac{1}{2}h^{\alpha\beta}h^{\gamma\delta}{}_{;\alpha}h_{\gamma\delta;\beta} + 2h^{\alpha\beta}h^{\gamma\delta}{}_{;\alpha}h_{\beta\gamma;\delta} - h^{\alpha\beta}h^{\gamma}{}_{\gamma;\alpha}h^{\delta}{}_{\beta;\delta} - \frac{1}{2}h^{\alpha}{}_{\alpha}h^{\beta\gamma;\delta}h_{\beta\delta;\gamma}\right.$$
$$+ \frac{1}{4}h^{\alpha}{}_{\alpha}h^{\beta\gamma;\delta}h_{\beta\gamma;\delta} - h^{\alpha\beta}h^{\gamma}{}_{\gamma;\delta}h^{\delta}{}_{\alpha;\beta} + \frac{1}{2}h^{\alpha\beta}h^{\gamma}{}_{\gamma;\alpha}h^{\delta}{}_{\delta;\beta} - h^{\alpha\beta}h_{\alpha\beta;\gamma}h^{\gamma\delta}{}_{;\delta}$$
$$+ \frac{1}{2}h^{\alpha}{}_{\alpha}h^{\beta}{}_{\beta;\gamma}h^{\gamma\delta}{}_{;\delta} + h^{\alpha\beta}h_{\alpha\beta;\gamma}h_{\delta}{}^{\delta;\gamma} + \frac{1}{4}h^{\alpha}{}_{\alpha}h^{\beta}{}_{\beta;\gamma}h_{\delta}{}^{\delta;\gamma} - h^{\alpha\beta}h^{\gamma}{}_{\alpha;\delta}h_{\beta\gamma}{}^{;\delta}$$
$$+ h^{\alpha\beta}h^{\gamma}{}_{\alpha;\delta}h^{\delta}{}_{\beta;\gamma} + R_{\alpha\beta}(2h^{\alpha\gamma}h_{\gamma\delta}h^{\beta\delta} - h^{\gamma}{}_{\gamma}h^{\alpha\delta}h^{\beta}{}_{\delta} - \frac{1}{2}h^{\alpha\beta}h^{\gamma\delta}h_{\gamma\delta}$$
$$\left. + \frac{1}{4}h^{\alpha\beta}h^{\gamma}{}_{\gamma}h^{\delta}{}_{\delta}) + R(-\frac{1}{3}h^{\alpha\beta}h_{\beta\gamma}h^{\gamma}{}_{\alpha} + \frac{1}{4}h^{\alpha}{}_{\alpha}h^{\beta\gamma}h_{\beta\gamma} - \frac{1}{24}h^{\alpha}{}_{\alpha}h^{\beta}{}_{\beta}h^{\gamma}{}_{\gamma})\right\}$$

even in flat space, the corresponding vertex has about 100 terms!

quartic order

$$
\begin{aligned}
L_4 = \sqrt{-g}\Big\{&(h^\alpha{}_\alpha h^\beta{}_\beta - 2h^{\alpha\beta}h_{\alpha\beta})\Big(\frac{1}{16}h^{\gamma\delta;\sigma}h_{\gamma\delta;\sigma} - \frac{1}{8}h^{\gamma\delta;\sigma}h_{\gamma\sigma;\delta} + \frac{1}{8}h^\gamma{}_{\gamma;\delta}h^{\delta\sigma}{}_{;\sigma} \\
&- \frac{1}{16}h^\gamma{}_{\gamma;\delta}h_\sigma{}^{\sigma;\delta}\Big) + h^\alpha{}_\alpha h^{\beta\gamma}\Big(-\frac{1}{2}h_{\beta\gamma;\delta}h^{\delta\sigma}{}_{;\sigma} + \frac{1}{2}h_{\beta\gamma;\delta}h_\sigma{}^{\sigma;\delta} - \frac{1}{2}h^\delta{}_{\delta;\beta}h^\sigma{}_{\sigma;\gamma} \\
&+ \frac{1}{4}h^\delta{}_{\delta;\beta}h^\sigma{}_{\sigma;\gamma} + h^\delta{}_{\beta;\sigma}h^\sigma{}_{\delta;\gamma} - \frac{1}{4}h^{\delta\sigma}{}_{;\beta}h_{\delta\sigma;\gamma} - \frac{1}{2}h^\delta{}_{\beta;\sigma}h_{\delta\gamma}{}^{;\sigma} - \frac{1}{2}h^\delta{}_{\delta;\sigma}h^\sigma{}_{\beta;\gamma} \\
&+ \frac{1}{2}h_{\beta\delta;\sigma}h_\gamma{}^{\sigma;\delta}\Big) + h^\alpha{}_\beta h^{\beta\gamma}\Big(h^\delta{}_{\delta;\sigma}h^\sigma{}_{\alpha;\gamma} - h_{\alpha\gamma;\delta}h_\sigma{}^{\sigma;\delta} + \frac{1}{2}h^{\delta\sigma}{}_{;\alpha}h_{\delta\sigma;\gamma} \\
&- h^\delta{}_{\alpha;\sigma}h^\sigma{}_{\gamma;\delta} - 2h^\delta{}_{\alpha;\sigma}h^\sigma{}_{\delta;\gamma} + h_{\alpha\gamma;\delta}h^{\delta\sigma}{}_{;\sigma} + h^\delta{}_{\delta;\alpha}h^\sigma{}_{\gamma;\sigma} - \frac{1}{2}h^\delta{}_{\delta;\alpha}h^\sigma{}_{\sigma;\gamma} \\
&+ h^\delta{}_{\alpha;\sigma}h_{\gamma\delta}{}^{;\sigma}\Big) + h^{\alpha\gamma}h^{\beta\delta}\Big(h_{\alpha\gamma;\beta}h^\sigma{}_{\delta;\sigma} - h_{\alpha\gamma;\delta}h^\sigma{}_{\sigma;\beta} + \frac{1}{2}h_{\alpha\beta;\sigma}h_{\gamma\delta}{}^{;\sigma} \\
&- \frac{1}{2}h_{\alpha\gamma;\sigma}h_{\beta\delta}{}^{;\sigma} + h^\sigma{}_{\alpha;\beta}h_{\gamma\sigma;\delta} - h^\sigma{}_{\alpha;\beta}h_{\delta\sigma;\gamma} + h_{\alpha\beta;\delta}h^\sigma{}_{\sigma;\gamma} - 2h^\sigma{}_{\alpha;\beta}h_{\delta\gamma;\sigma} \\
&+ h_{\alpha\gamma;\sigma}h^\sigma{}_{\beta;\delta}\Big) + R_{\alpha\beta}\Big(-2h^{\alpha\gamma}h_{\gamma\delta}h^{\delta\sigma}h_\sigma{}^\beta + h^\gamma{}_\gamma h^{\alpha\delta}h_{\delta\sigma}h^{\sigma\beta} + \frac{1}{2}h^{\alpha\gamma}h_\gamma{}^\beta h^{\delta\sigma}h_{\delta\sigma} \\
&- \frac{1}{4}h^{\alpha\gamma}h_\gamma{}^\beta h^\delta{}_\delta h^\sigma{}_\sigma + \frac{1}{3}h^{\alpha\beta}h^{\gamma\delta}h_{\delta\sigma}h^\sigma{}_\gamma - \frac{1}{4}h^{\alpha\beta}h^\gamma{}_\gamma h^{\delta\sigma}h_{\delta\sigma} + \frac{1}{24}h^{\alpha\beta}h^\gamma{}_\gamma h^\delta{}_\delta h^\sigma{}_\sigma\Big) \\
&+ R\Big(-\frac{1}{192}h^\alpha{}_\alpha h^\beta{}_\beta h^\gamma{}_\gamma h^\delta{}_\delta + \frac{1}{16}h^\alpha{}_\alpha h^\beta{}_\beta h^{\gamma\delta}h_{\gamma\delta} + \frac{1}{4}h^{\alpha\beta}h_{\beta\gamma}h^{\gamma\delta}h_{\delta\alpha} \\
&- \frac{1}{16}h^{\alpha\beta}h_{\alpha\beta}h^{\gamma\delta}h_{\gamma\delta} - \frac{1}{6}h^\alpha{}_\alpha h^{\beta\gamma}h_{\gamma\delta}h^\delta{}_\beta\Big)\Big\}
\end{aligned}
$$

Imagine having to do calculations with these interaction vertices!

Summary of the gauge-theoretic formulation

- Uses connections instead of metrics to describe gravitons

At the linearized level $S_+^3 \otimes S_-$ instead of $S_+^2 \otimes S_-^2$

parity invariance non-manifest!

- Simpler than the metric-based GR in many aspects

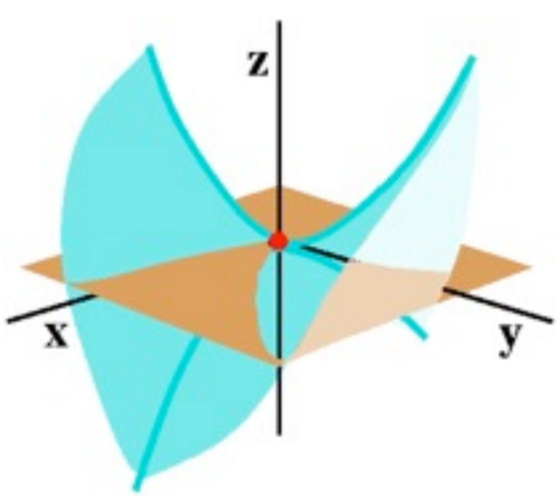
hermiticity non-manifest!

vertices are much simpler in this formulation

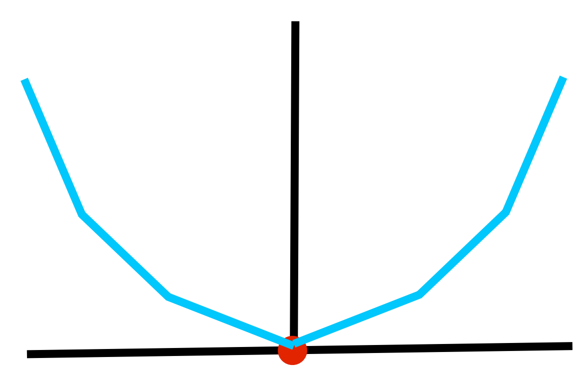
functional in the space of conformal classes rather than full metrics

convex action functional

$S_{\rm EH}[g]$



space of metrics



$S[A]$

space of conformal metrics = SU(2) connections/gauge

Summary of results on the pure connection formulation (so far)

- Description of the linearized theory, including the mode decomposition and the reality conditions

  Gianluca Delfino, KK, Carlos Scarinci arXiv:1205.7045

- Feynman rules, graviton-graviton scattering amplitudes

  Gianluca Delfino, KK, Carlos Scarinci arXiv:1210.6215

- One-loop computation (instanton background)

  Kai Groh, KK, Christian Steinwachs arXiv:1304.6946

The pure connection formulation of GR suggests generalizations that are difficult to imagine in the metric formulation

Diffeomorphism invariant gauge theories

Can define a gauge and diffeomorphism invariant action

$$F = dA + (1/2)[A, A]$$

$$\boxed{S[A] = \int f(F \wedge F)}$$

no dimensionful coupling constants!

Field equations: $d_A B = 0$

where $B = \frac{\partial f}{\partial X} F$ and $X = F \wedge F$

Second-order (non-linear) PDE's

compare Yang-Mills equations: $d_A B = 0$

where $B = {}^* F$

$*$ - encodes the metric

Dynamically non-trivial theory with 2n-4 propagating DOF

apart from the single point $f_{top} = \mathrm{Tr}(F \wedge F)$

Gauge symmetries:

$\delta_\phi A = d_A \phi$ gauge rotations

$\delta_\xi A = \iota_\xi F$ diffeomorphisms

The simplest non-trivial theory:

G=SU(2) - gravity

(interacting massless spin 2 particles)

Define the metric by:

$$\mathrm{Span}\{F(A)\} = \Lambda^+ M \qquad\qquad (\mathrm{vol}) \sim f(F\wedge F)$$

The functional is just the volume: $\boxed{S[A] \sim \mathrm{Vol}(M)}$

The linearization (around de Sitter) is the same for any f()

For any choice of f() - a theory of interacting massless spin 2 particles

specific f() - GR

Deformations of GR

All other choices of f() lead to different (from GR) interacting theories of massless spin 2 particles

The action, when expressed in metric terms, is of the form

$$S[g] = \frac{1}{2\kappa^2}\int \sqrt{-g}\left(R - 2\Lambda + \lambda_1 \kappa^4 C^3 + \sum_{n=2}^{\infty} \lambda_n \kappa^{2n+2} I_n(\nabla, C)\right)$$

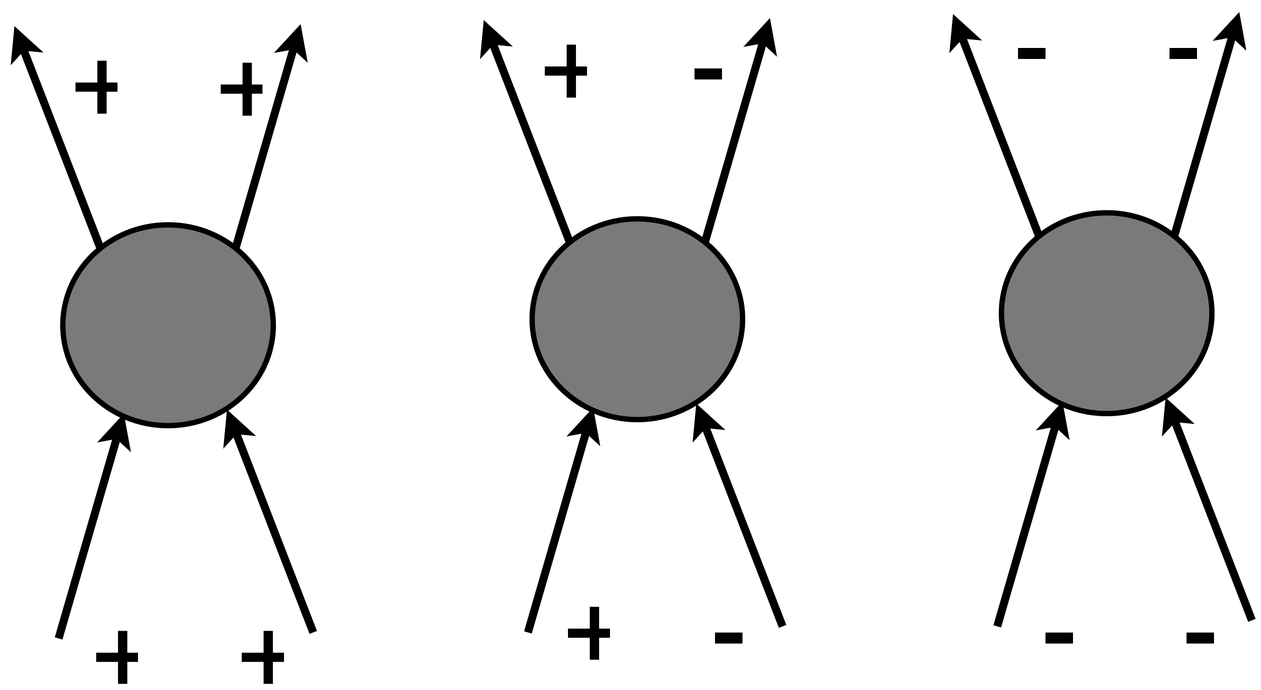
C - Weyl curvature

In general with parity violating terms:
A generic theory is not parity invariant!

Modified gravity theories with 2 propagating DOF - a very interesting object of study

Parity violation is quantified in scattering amplitudes

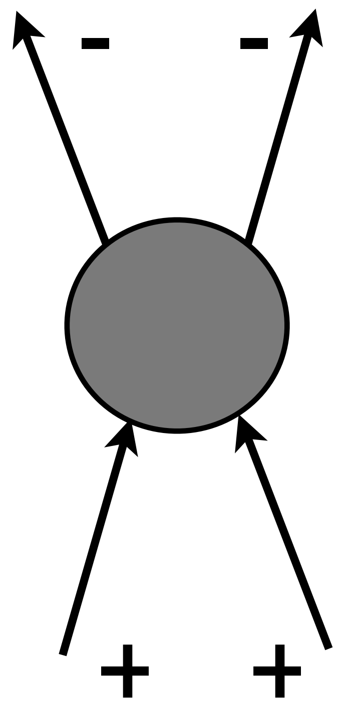
In GR only parity-preserving processes:



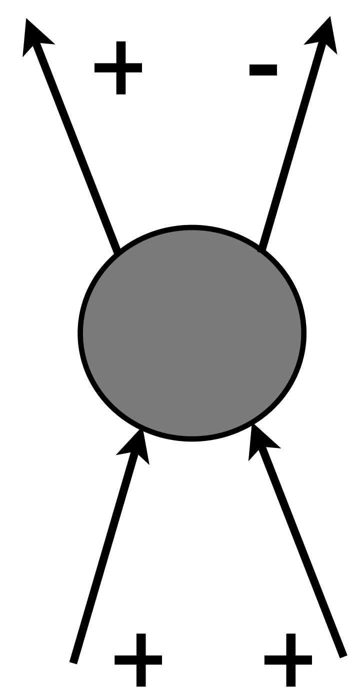
amplitude $$\mathcal{A} \sim \frac{1}{M_p^2}\frac{s^3}{tu} \sim \left(\frac{E}{M_p}\right)^2$$

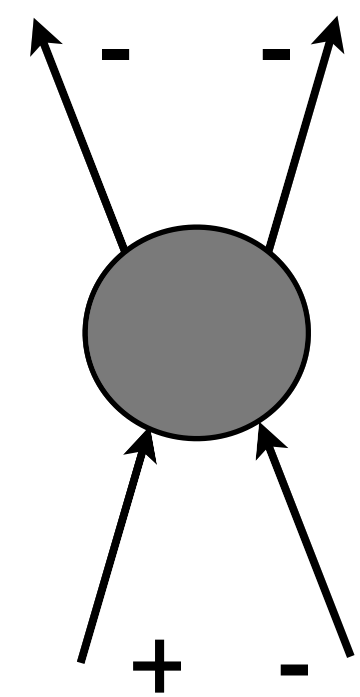
becomes larger than unity at Planck energies, cannot trust perturbation theory

In a general theory from our family parity-violating processes become allowed:



$$\mathcal{A} \sim \frac{s^4 + t^4 + u^4}{M_p^8} \sim \left(\frac{E}{M_p}\right)^8$$





$$\mathcal{A} \sim \frac{stu}{M_p^6} \sim \left(\frac{E}{M_p}\right)^6$$

A general theory likes negative helicity gravitons!

Can speculate that at high energies these processes will dominate and all gravitons will get converted into negative helicity ones (strongly coupled by the parity-preserving processes)

Quantum Theory Hopes

Remark: no dimensionful coupling constants in any of these gravitational theories

(negative) dimension coupling constant comes when expanded around a background

Non-renormalizable in the usual sense

Hope: the class of theories {all possible f()} is large enough to be closed under renormalization

$$\frac{\partial f(F \wedge F)}{\partial \log \mu} = \beta_f(F \wedge F)$$

I.e. physics at higher energies continues to be described by theories from the same family

= no new DOF appear at Planck scale, just the dynamics changes

The speculative RG flow

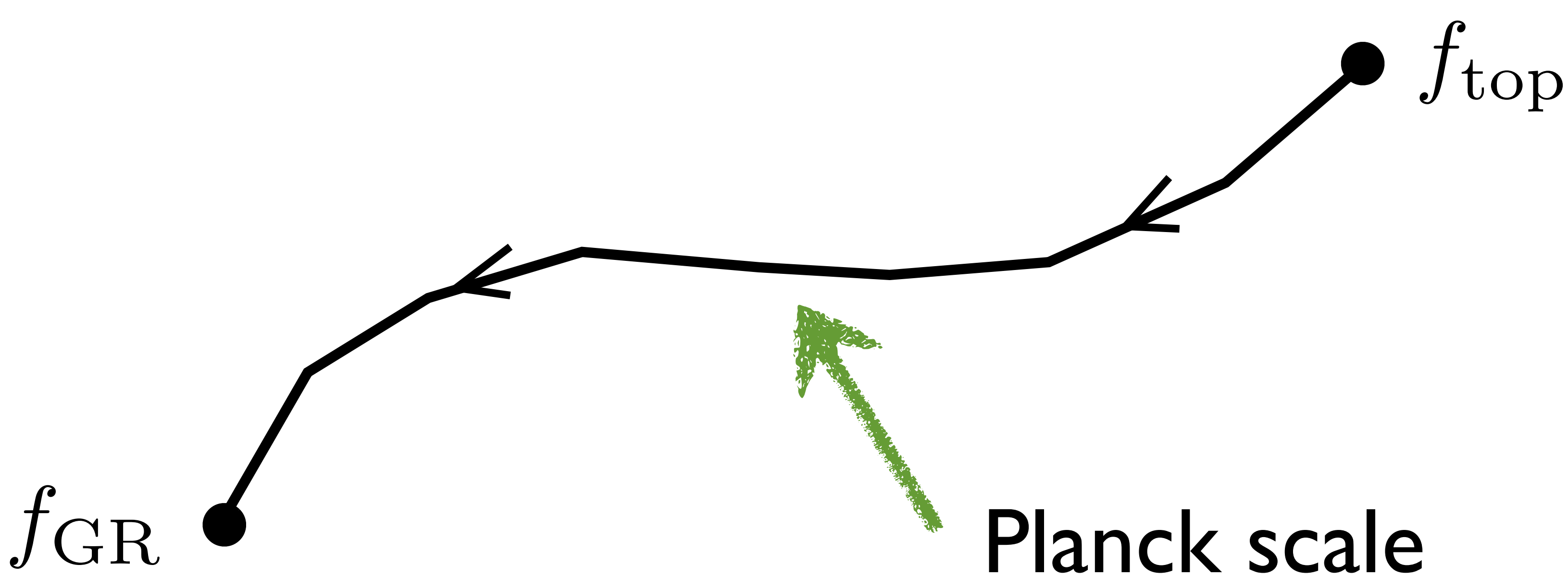
strongly coupled negative helicity gravitons at high energies

$\Rightarrow$ no propagating DOF ? $\Rightarrow$ topological theory ?

$$f_{\text{top}}(F \wedge F) = \text{Tr}(F \wedge F)$$

necessarily a fixed point of the RG flow

corresponds to a topological theory (no propagating DOF)



flow from very steep in IR towards very flat in UV potential

Another possible application: Unification

Consider $G \supset \mathrm{SU}(2)$

A class of backgrounds is classified by how SU(2) embeds into G

Generically, there is a part of G that commutes with the SU(2)

E.g. $$\mathrm{SU}(3) \ni \begin{pmatrix} \mathrm{SU}(2) & * \\ * & * \end{pmatrix}$$

The SU(2) part of the connection continues to describe gravitons

The part that commutes with SU(2) describes YM gauge fields

Summary:

- Dynamically non-trivial diffeomorphism invariant gauge theories
- The simplest non-trivial such theory G=SU(2) - gravity
- GR can be described in this language (on-shell equivalent only)

  $\Rightarrow$ possibly different quantum theory

- Computationally efficient alternative to the usual description (no propagating conformal mode even off-shell)
- Different from GR (parity-violating) theories of interacting massless spin 2 particles
- If this class of theories is closed under renormalization

  $\Rightarrow$ understanding of the gravitational RG flow
  description of the Planck scale physics

Open problems

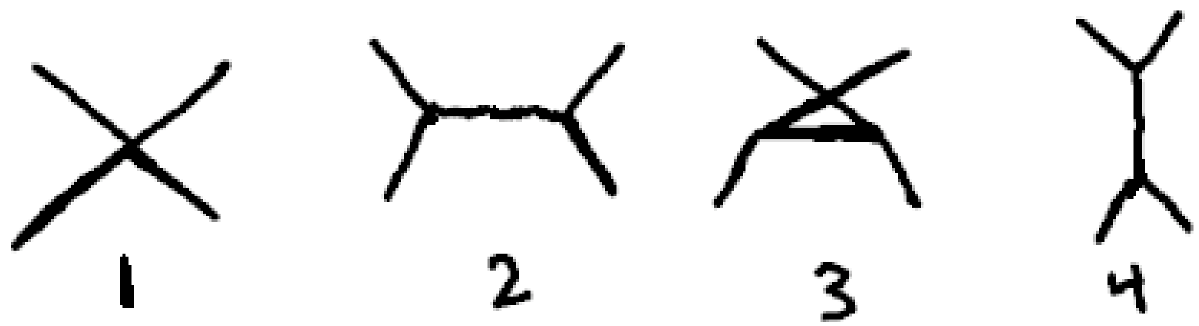
- Chiral, thus complex description. Unitarity?
- Coupling to matter?

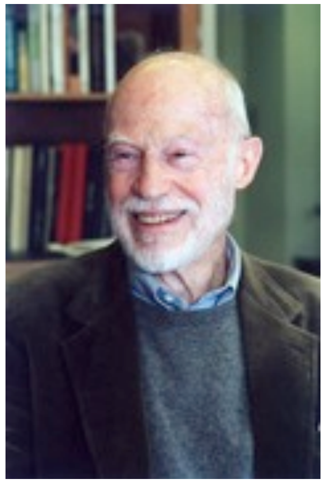
  Enlarging the gauge group - rather general types of matter coupled to gravity can be obtained. Fermions?
- Closedness under renormalization?

  Can these theories be as UV complete as Yang-Mills?

Still, they were done...

In 1963 I gave [Walter G. Wesley] a student of mine the problem of computing the cross section for a graviton-graviton scattering in tree approximation, for his Ph.D. thesis. The relevant diagrams are these:





Given the fact that the vertex function in diagram 1 contains over 175 terms and that the vertex functions in the remaining diagrams each contain 11 terms, leading to over 500 terms in all, you can see that this was not a trivial calculation, in the days before computers with algebraic manipulation capacities were available. And yet the final results were ridiculously simple. The cross section for scattering in the center-of-mass frame, of gravitons having opposite helicities, is

$$d\sigma/d\Omega = 4G^2E^2\cos^{12}\tfrac{1}{2}\theta/\sin^4\tfrac{1}{2}\theta$$

where G is the gravity constant and E is the energy.

From: Bryce DeWitt
arXiv:0805.2935
Quantum Gravity, Yesterday and Today

We now know that computing Feynman diagrams is not the simplest approach to the problem

Using the spinor helicity methods, the computation becomes doable

Using BCFW on-shell technology, the calculation becomes a homework exercise

Still, having a simpler off-shell description is important