School of Mathematical Sciences

G14TNS Theoretical Neuroscience

Problem sheet 1

1. Find the stationary points and determine their stability for the following examples. In each case sketch a bifurcation diagram and find the bifurcation values of the parameter.

   (a) $\dot{x} = x(\mu - x^2)(\mu - 2x^2)$

   (b) $\dot{x} = \mu - x^2 + 4x^4$

2. Consider
$$\dot{x} = x - y - x(x^2 + 5y^2), \quad \dot{y} = x + y - y(x^2 + y^2)$$

   (a) Classify the stability type of the fixed point at the origin.

   (b) Rewrite the system in polar coordinates using $r\dot{r} = x\dot{x} + y\dot{y}$ and $\dot{\theta} = (x\dot{y} - y\dot{x})/r$.

   (c) Determine the circle of maximum radius r_1 centred on the origin such that all trajectories have a radially outward component on it.

   (d) Determine the circle of minimum radius r_2 centred on the origin such that all trajectories have a radially inward component on it.

   (e) Prove that the system has a limit cycle somewhere in the trapping region $r_1 \leq r \leq r_2$.

3. Consider a single neuron with feedback
$$\tau\dot{x} = -x + f(x), \qquad f(x) = \tanh \beta x$$

   (a) Show that
$$f'(x) = \beta(1 - f^2(x))$$
   What is the resulting form of f in the limit $\beta \to \infty$?

   (b) Show that
$$f^{-1}(x) = -\frac{1}{2\beta} \ln \frac{1-x}{1+x}$$
   and sketch the function $f^{-1}(x)$.

   (c) Determine the fixed points of the model graphically.

   (d) Sketch a bifurcation diagram which shows how the stability of solutions varies with β.

4. Consider the bistable single neuron model
$$\frac{du}{dt} = f(u), \qquad f(u) = u(1-u)(u-a)$$
   where $0 < a < 1$.

   (a) Sketch the function $f(u)$ for the two cases $0 < a < 1/2$ and $1/2 < a < 1$.

   (b) Determine the stability at the fixed points $u = 0, a, 1$.

(c) Define a potential function $V(u)$ according to $f(u) = -\partial V/\partial u$ and $V(0) = 0$. Using the chain-rule show that $V(u(t))$ is a decreasing function of time. Sketch $V(u)$ for $0 < a < 1/2$ and for $1/2 < a < 1$ and indicate the locations of the stable and unstable fixed points.

S Coombes 2008