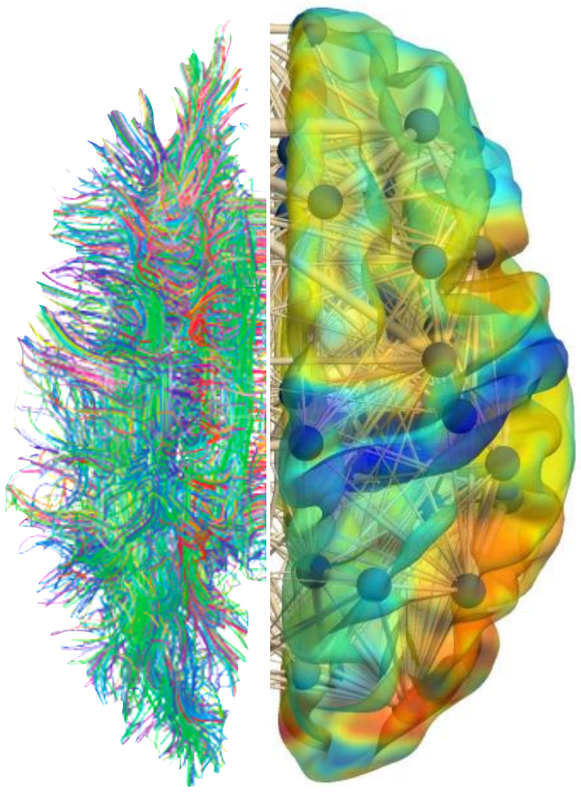
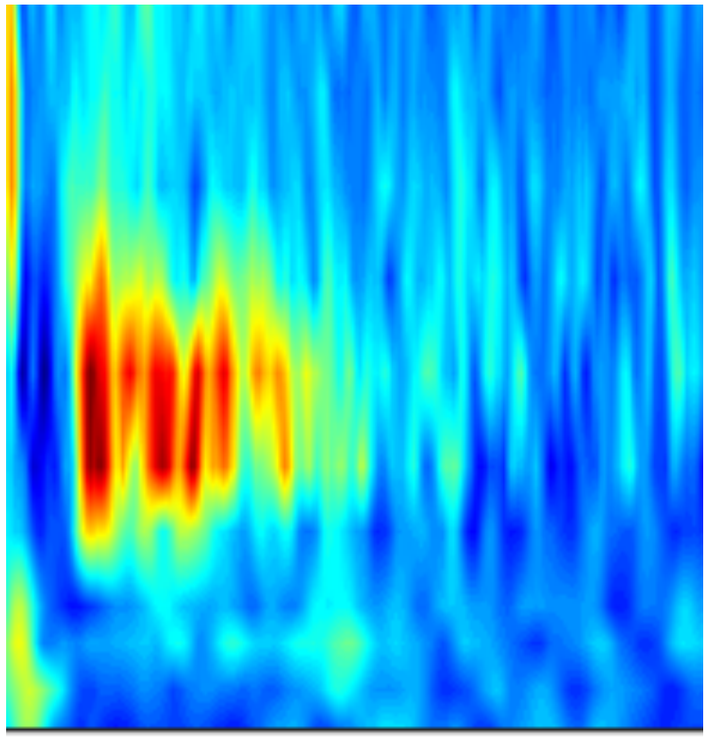
Modelling large scale brain dynamics



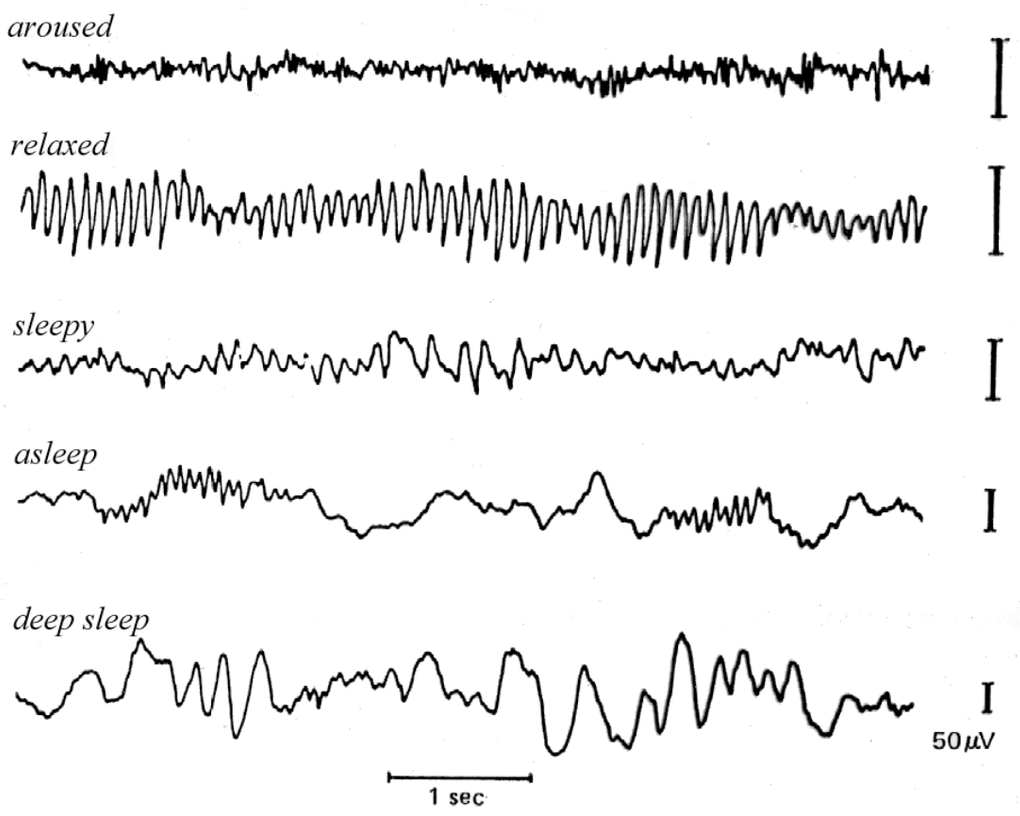


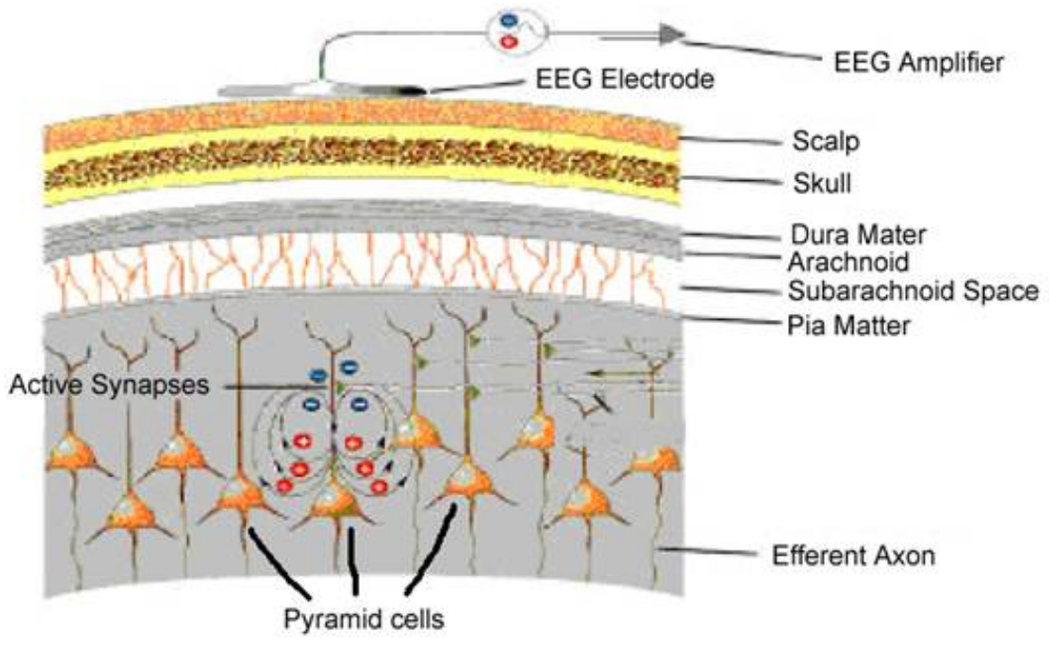
University of Nottingham
UK | CHINA | MALAYSIA

Steve Coombes

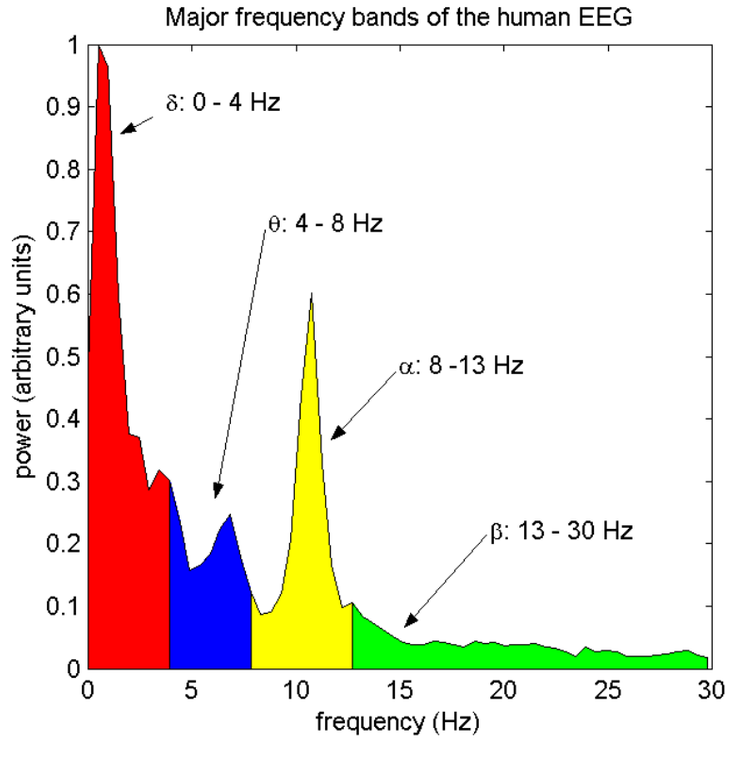
CMMB
Centre for Mathematical Medicine and Biology

Electroencephalogram (EEG) power spectrum

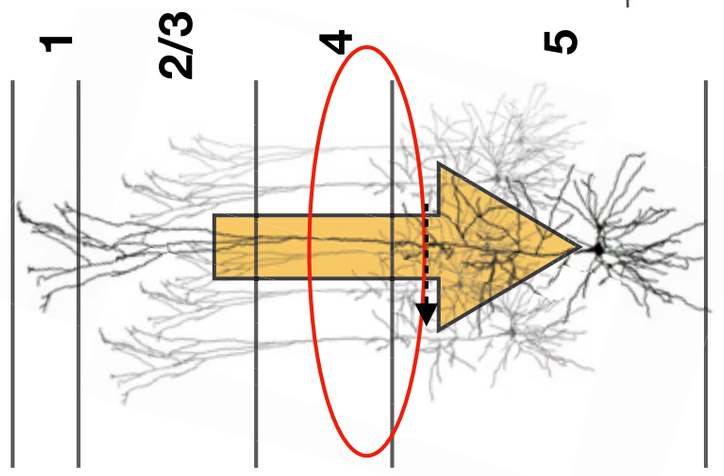


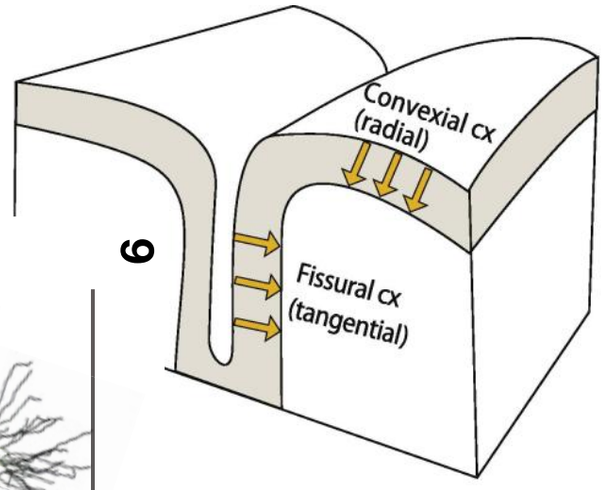


EEG records the activity of
~ 10^5 pyramidal neurons.

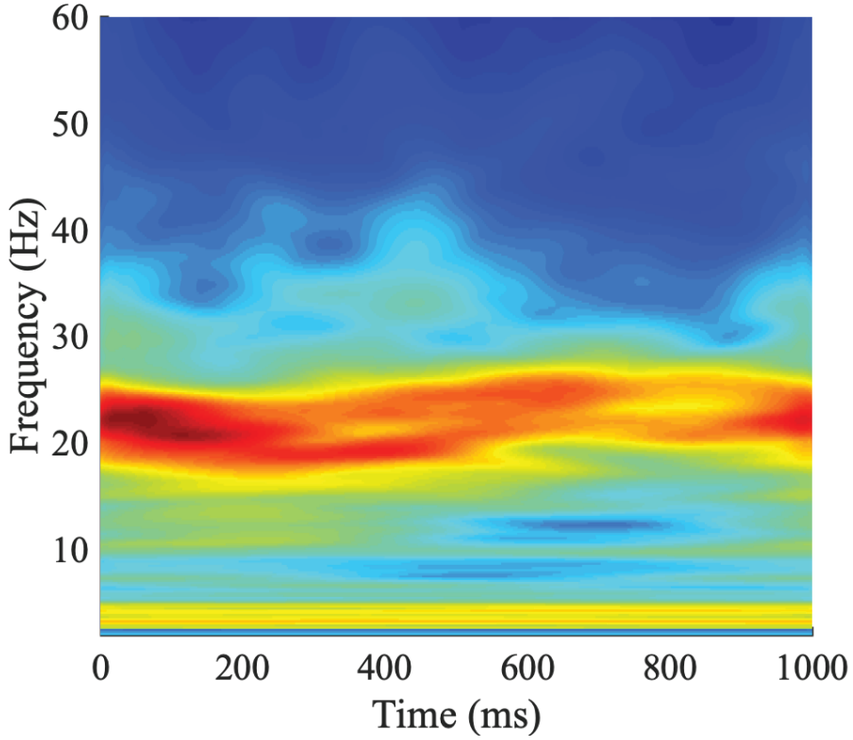


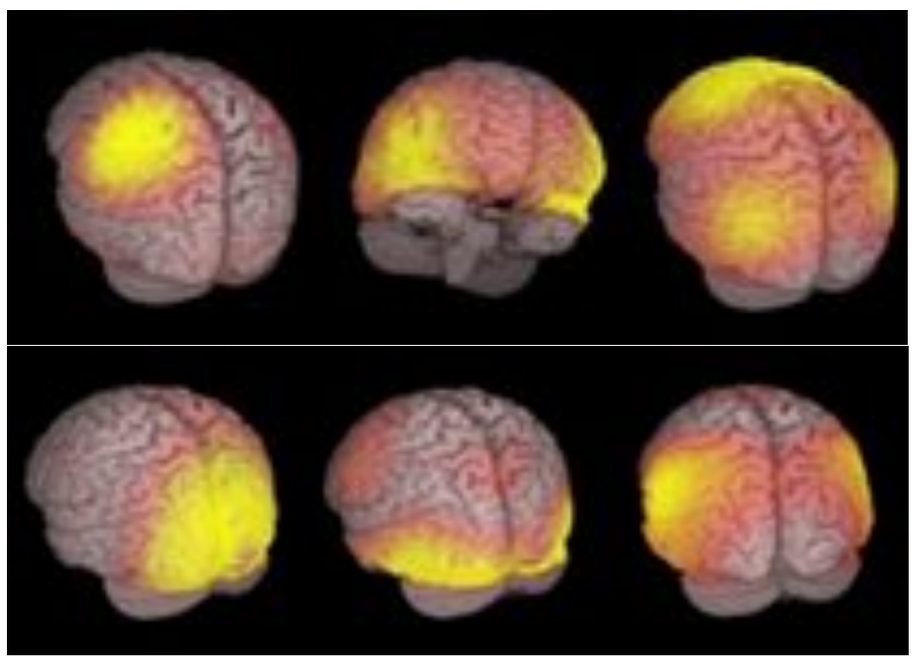
MEG





MEG records the activity of
~ 10^5 pyramidal neurons

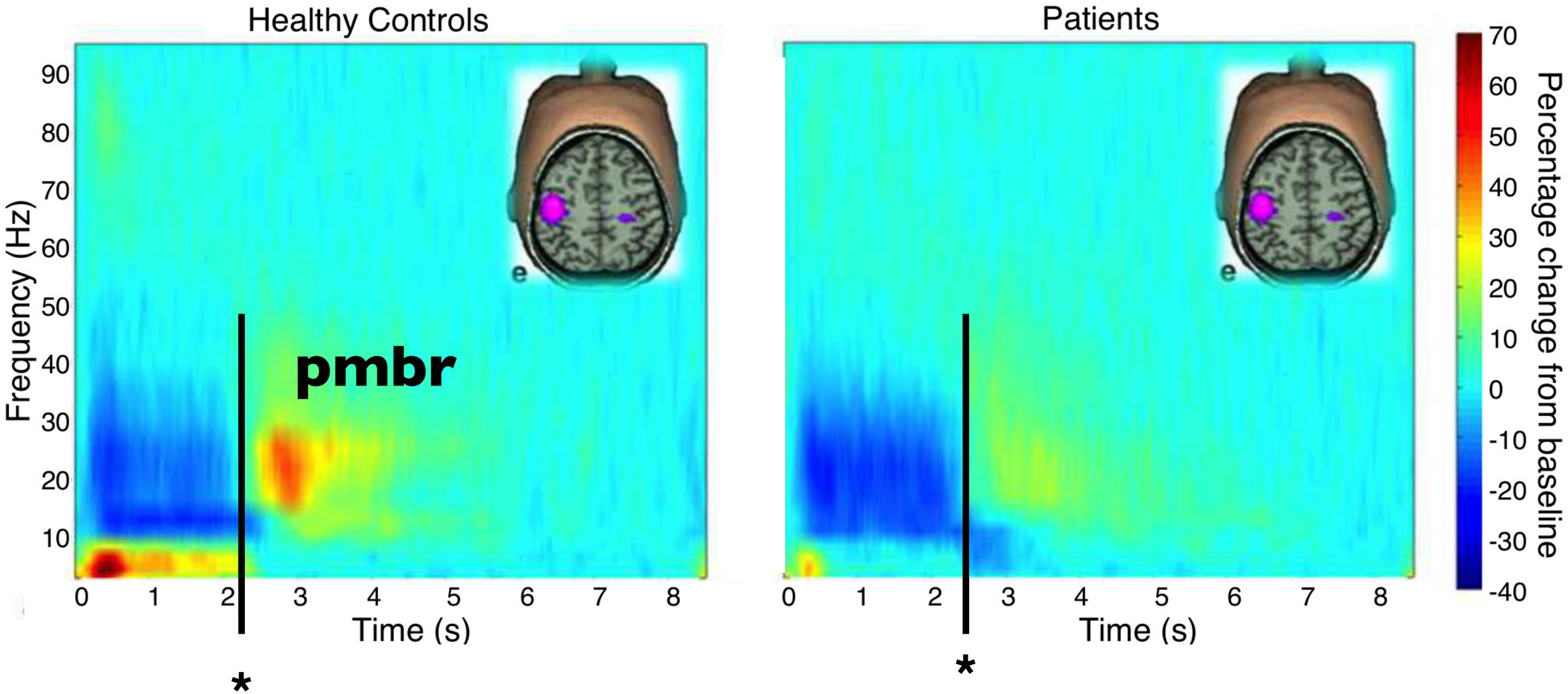




oscillatory brain modes, large scale distributed brain networks, pattern connectivity, cortical micro states, predictive coding, … [Magnetoencephalography in Cognitive Neuroscience: A Primer, 2019 Neuron, Joachim Gross]

Localised β rebound?

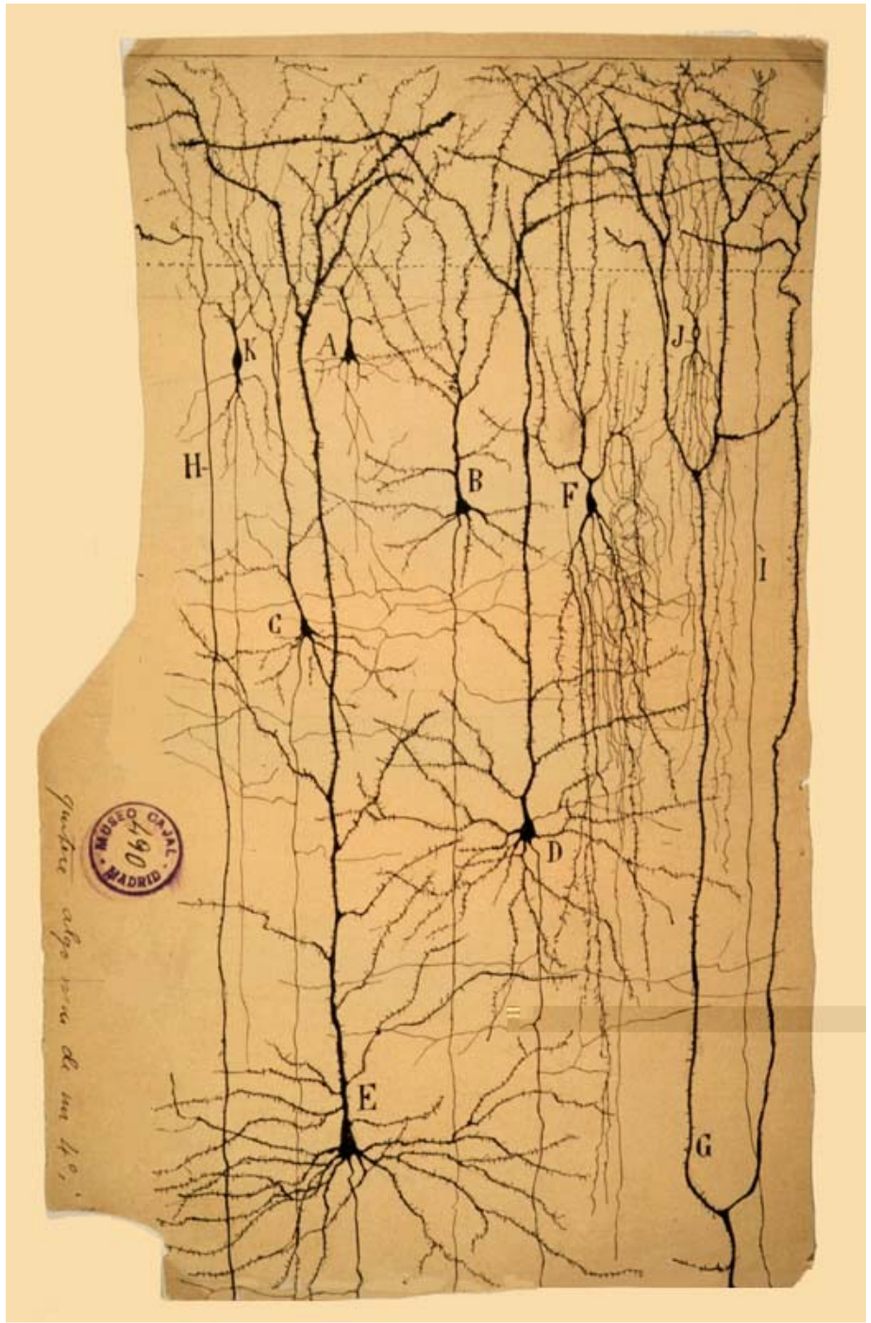
S E Robson, **M J Brookes**, …, **P F Liddle**, and P G Morris. Abnormal visuomotor processing in schizophrenia. NeuroImage: Clinical, 12:869 – 878, 2016.



* Movement termination

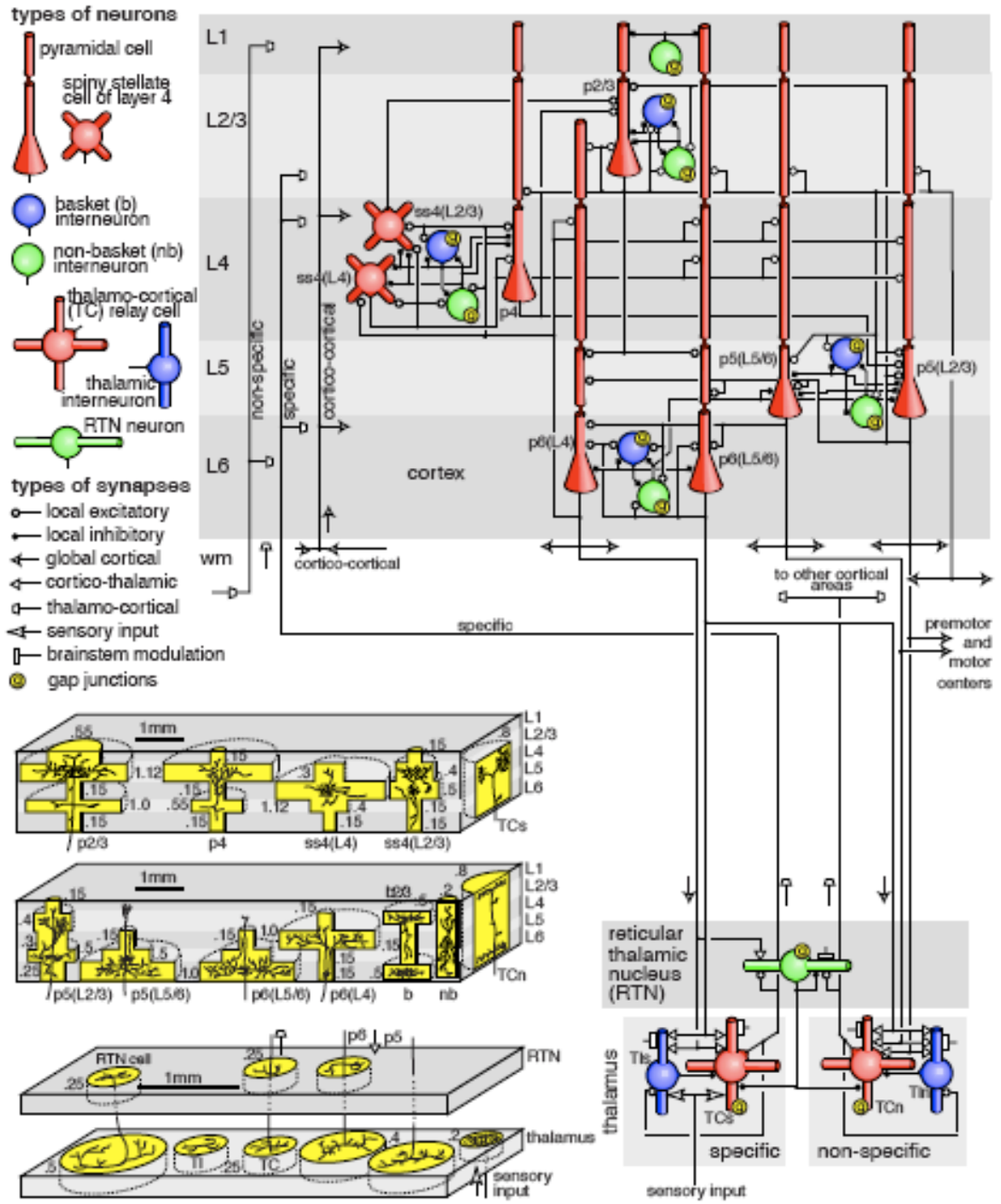
ERD/ERS

Cortical modelling



Santiago Ramón y Cajal

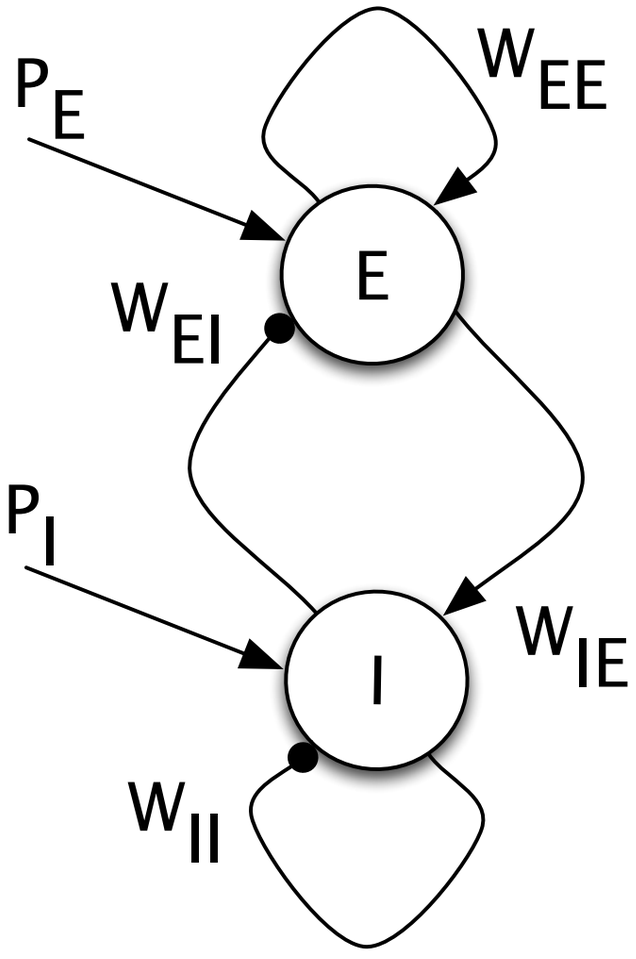
1900 Golgi's *black reaction*

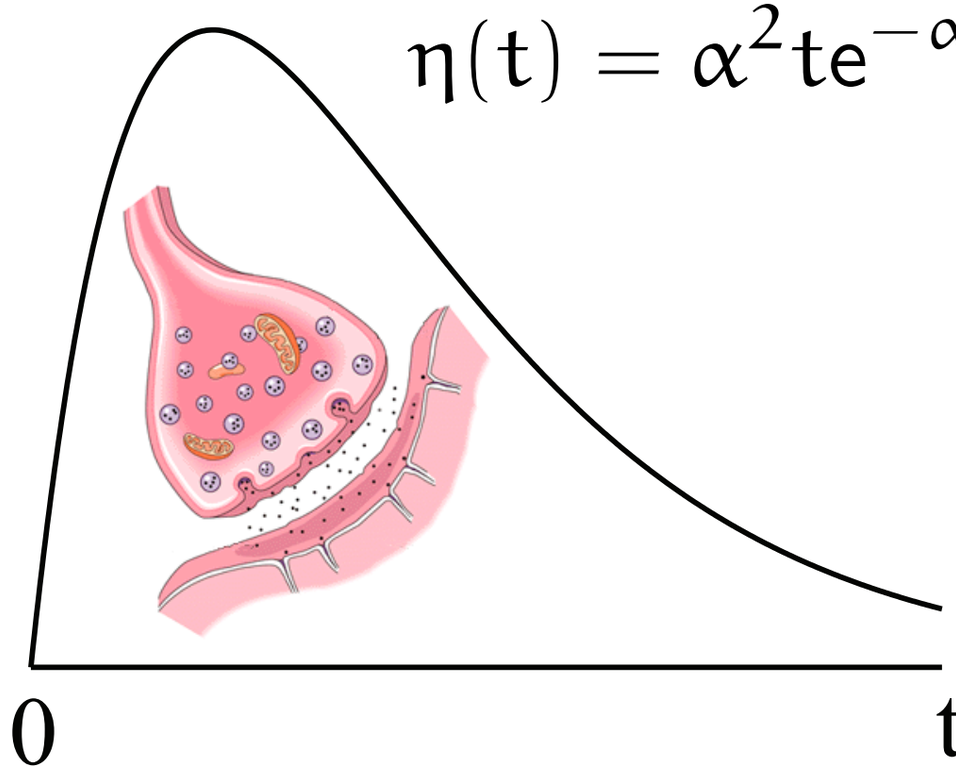


Eugene Izhikevich

2008

Traditional (phenomenological) approaches





$$Q\eta = \delta$$

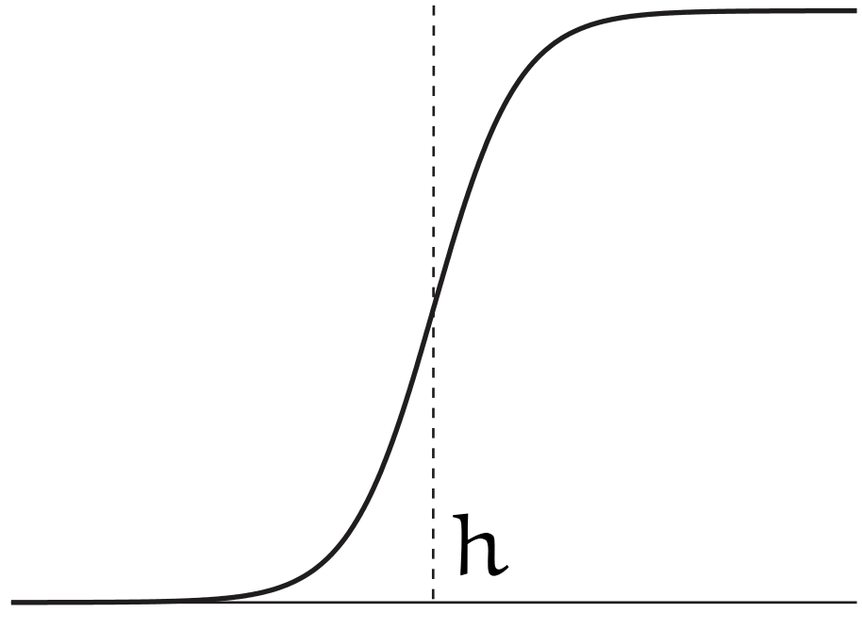
$$Q = \left(1 + \frac{1}{\alpha}\frac{d}{dt}\right)^2$$

or

$$Qg = f$$

$$g = \eta * f$$

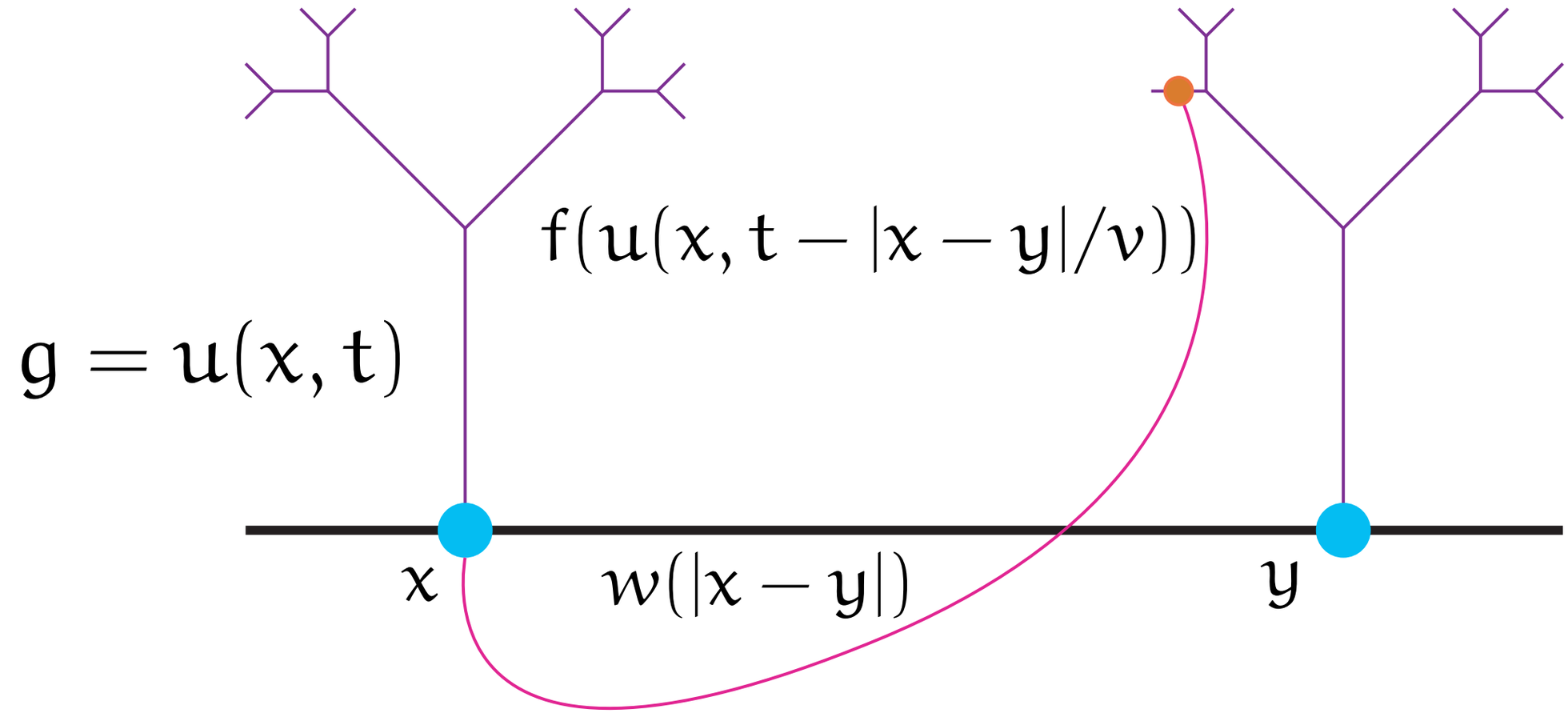
Firing rate activity $f(E)$



Spatially extended models

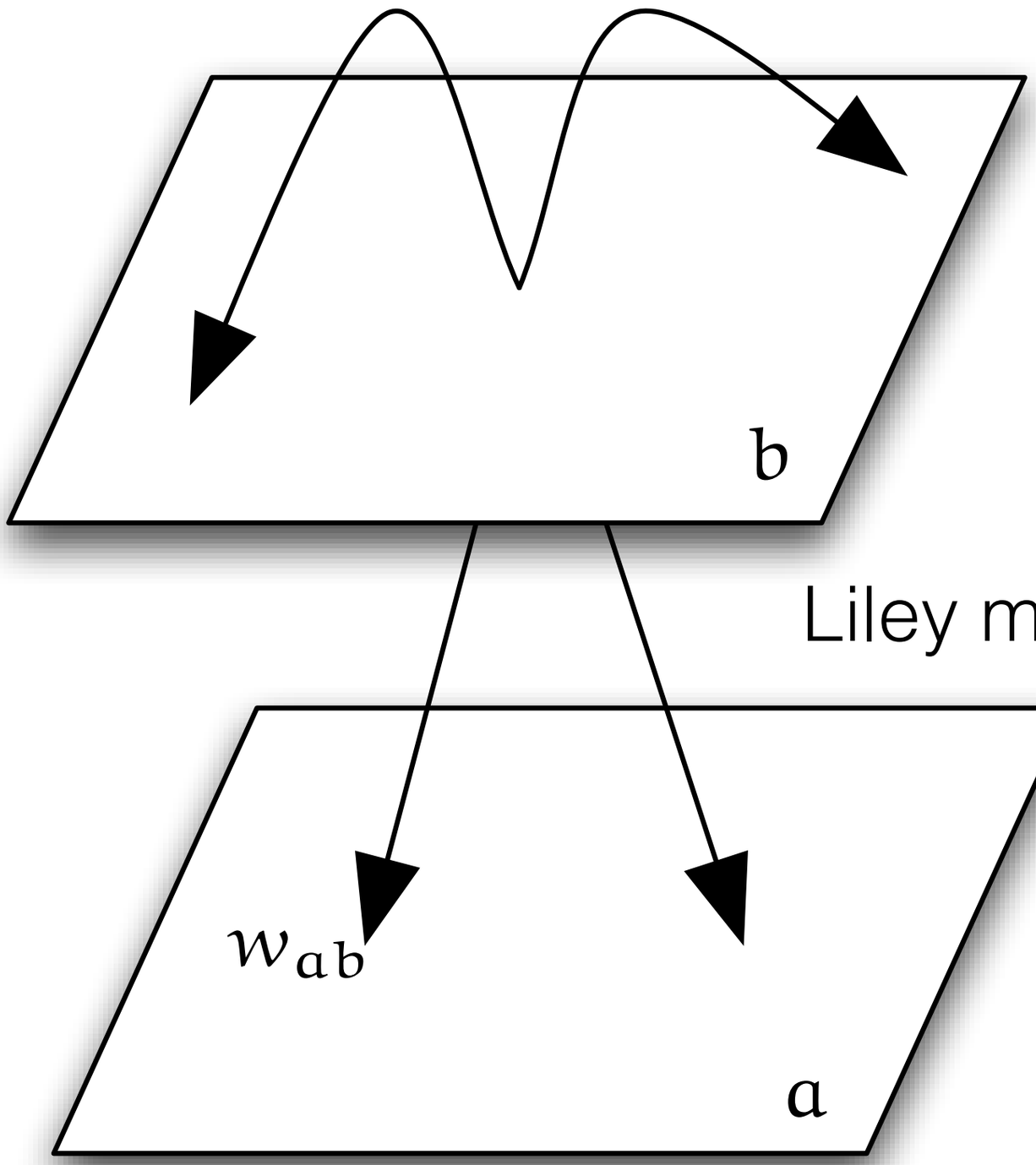
$$g = w \otimes \eta * f$$

Simplest neural field model: Wilson-Cowan ('72), Amari ('77)

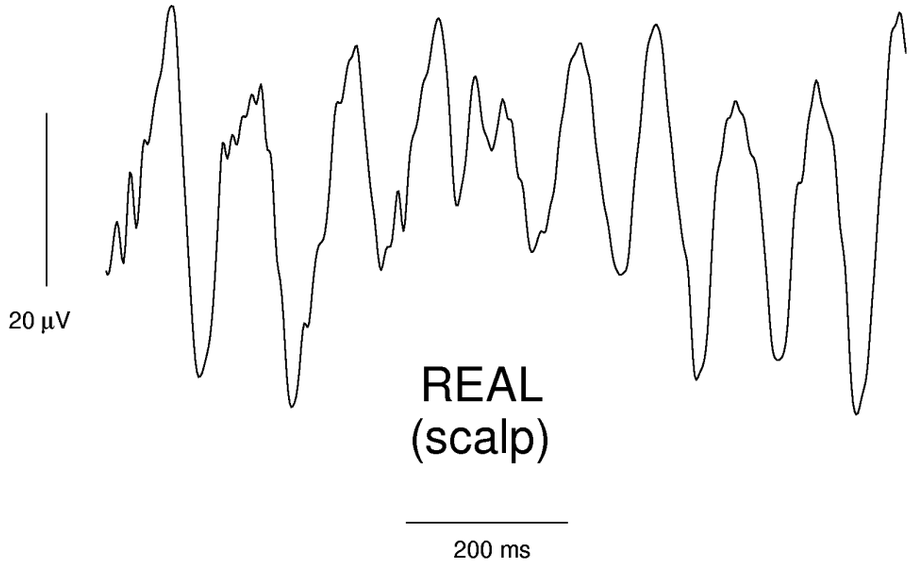


$$u(x,t) = \int_{-\infty}^{\infty} dy w(|x-y|) \int_0^t ds \eta(t-s) f \circ u(y, s - |x-y|/v)$$

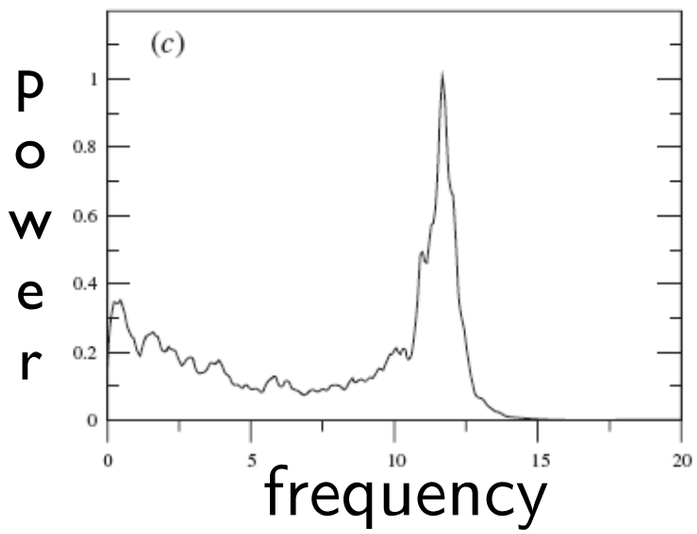
2D layers



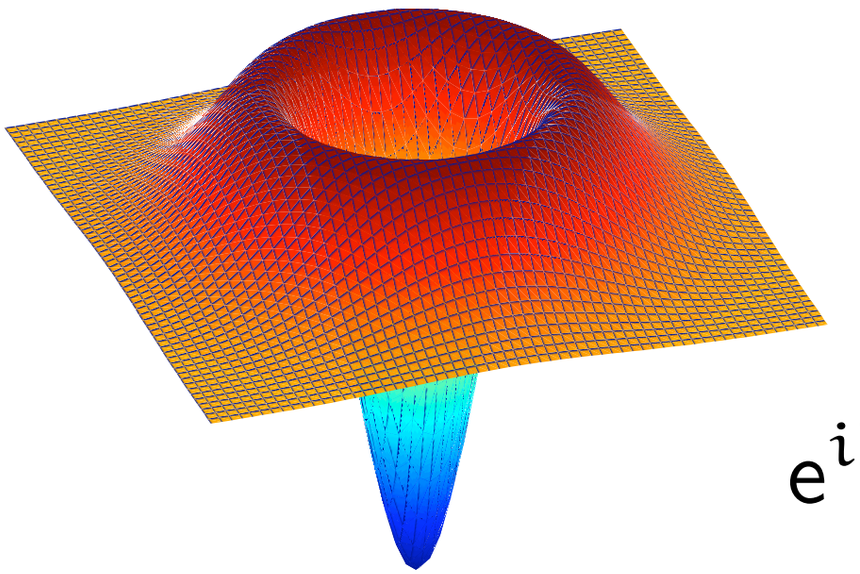
Liley model



SIMULATED
(cortex)



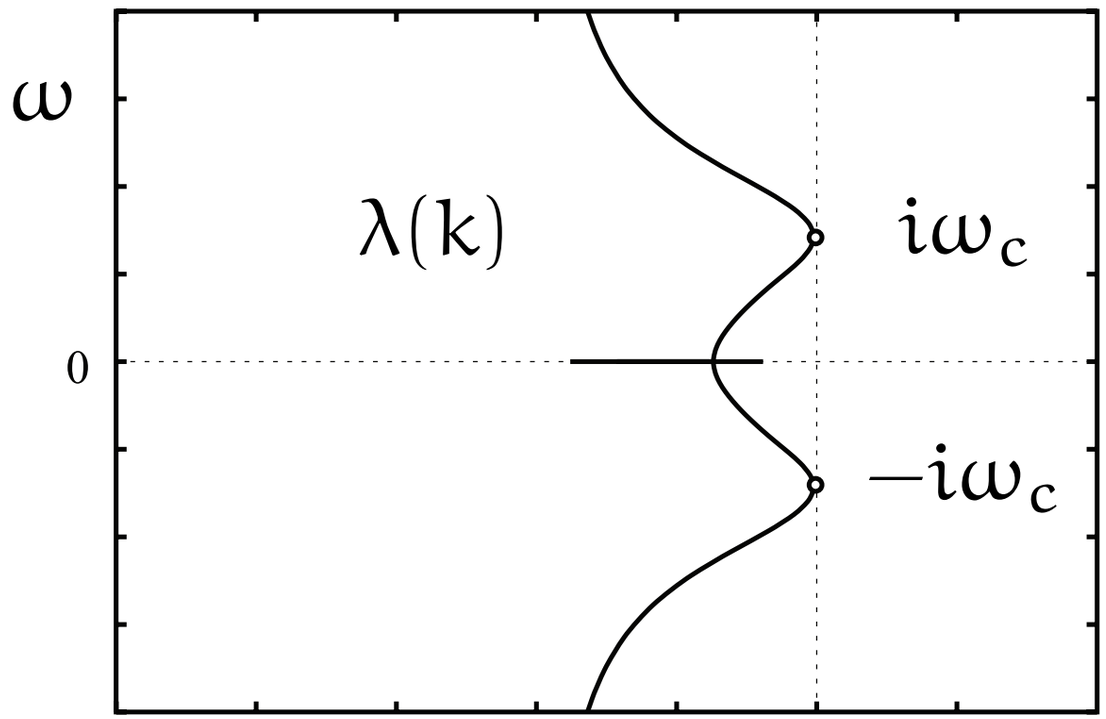
Turing instability analysis

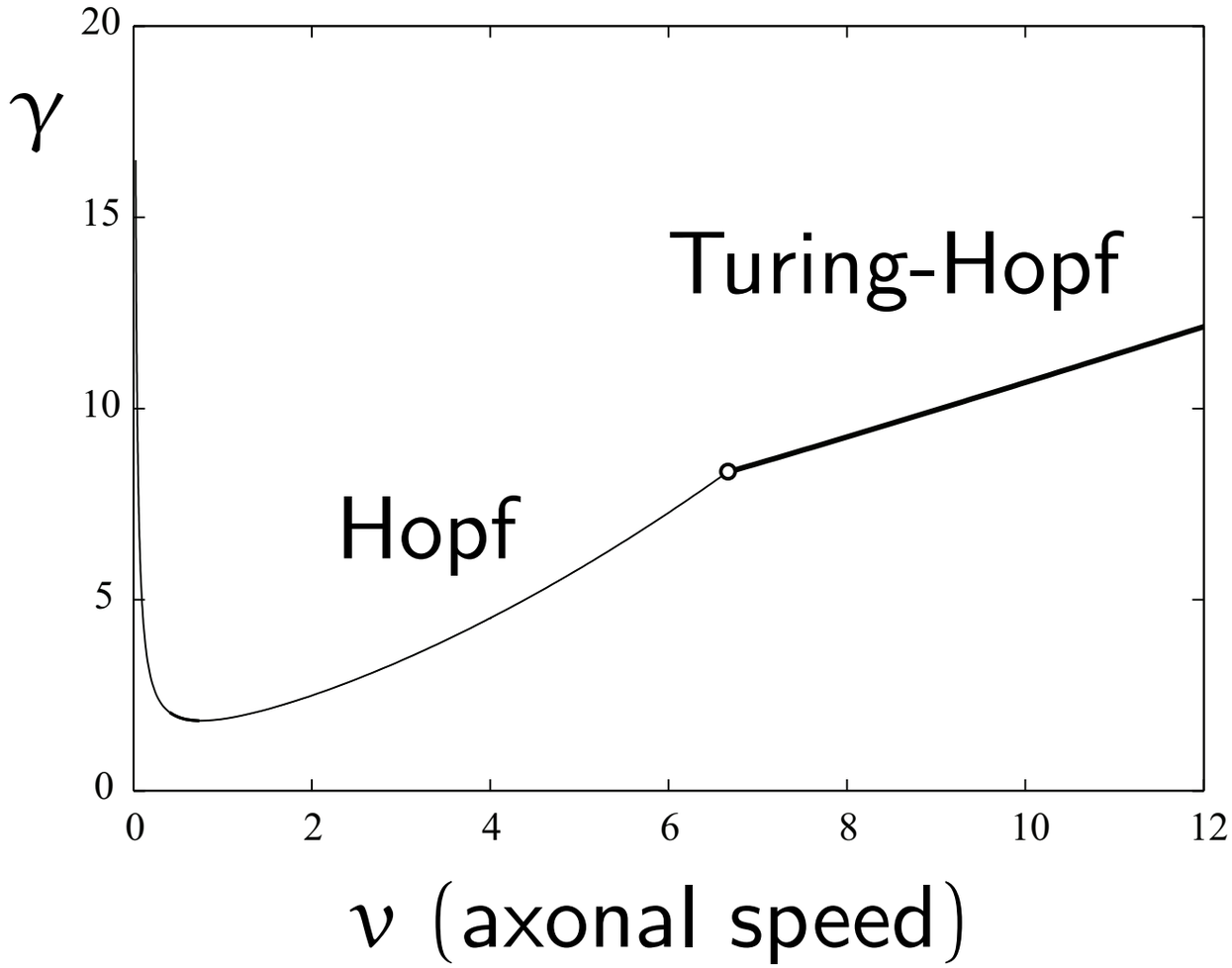


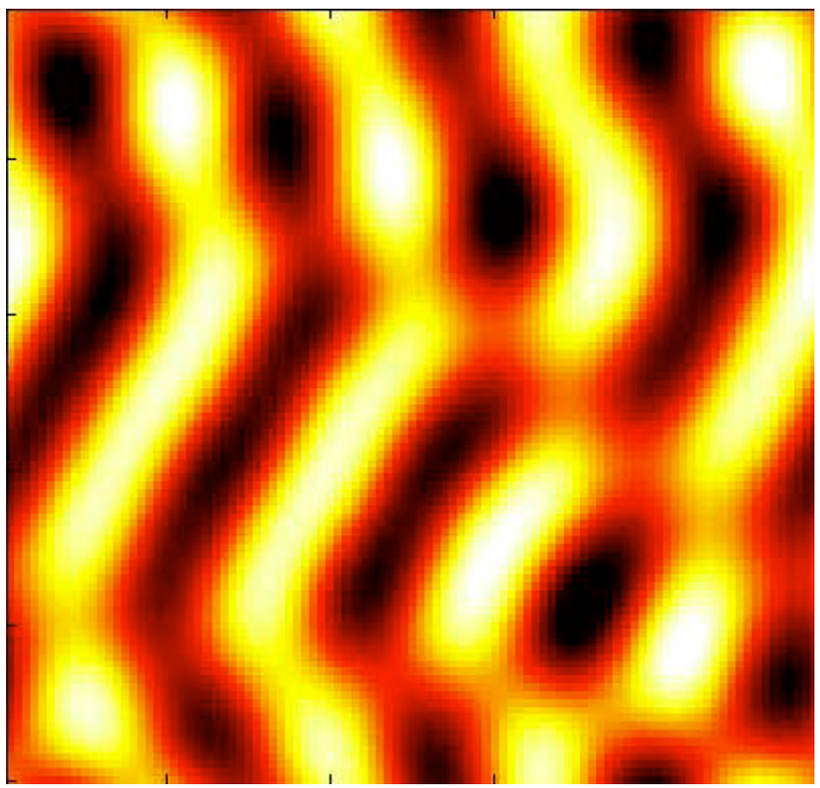
$$e^{i\mathbf{k}\cdot\mathbf{r}}e^{\lambda t}$$

E layer and I layer

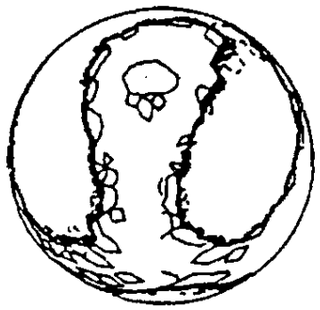
$$\lambda = \nu + i\omega$$



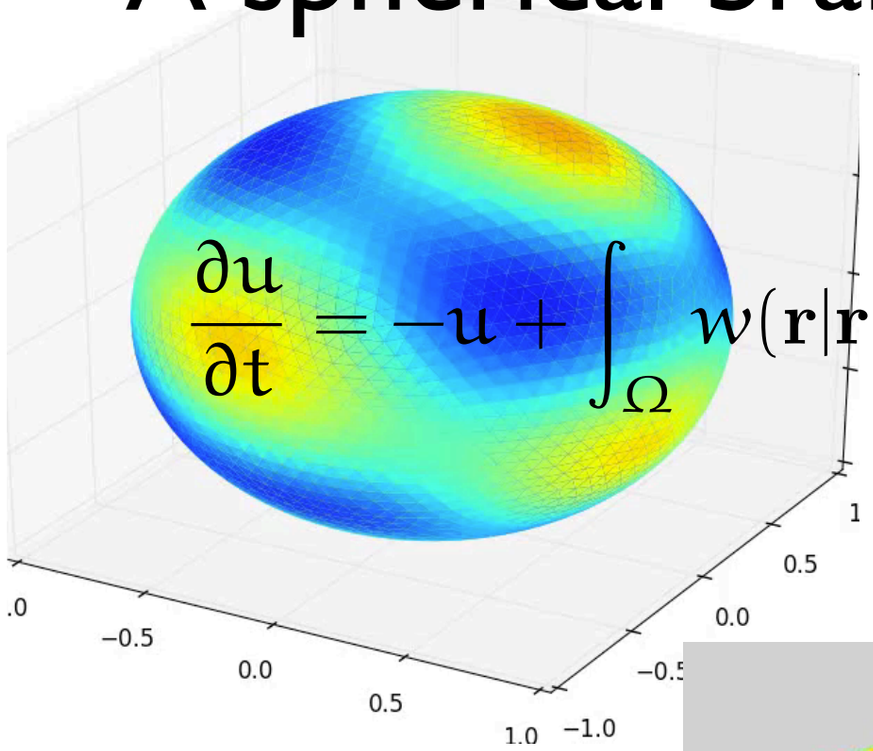


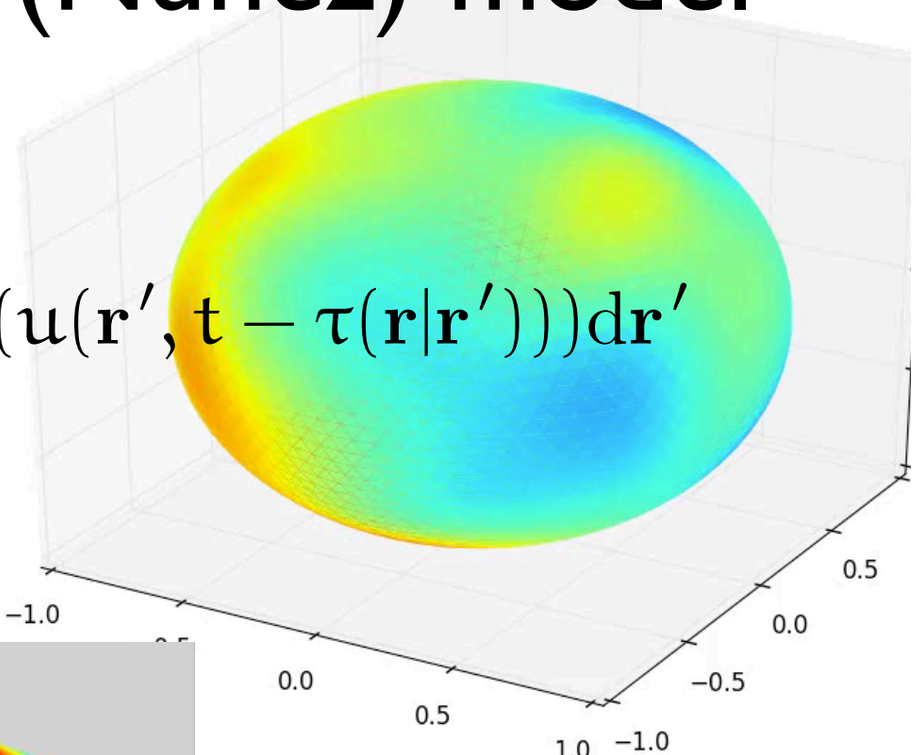


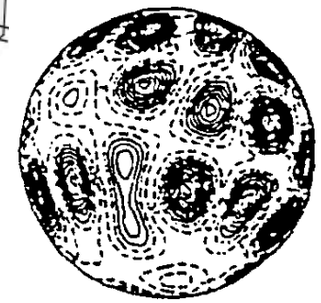
A spherical brain (Nunez) model

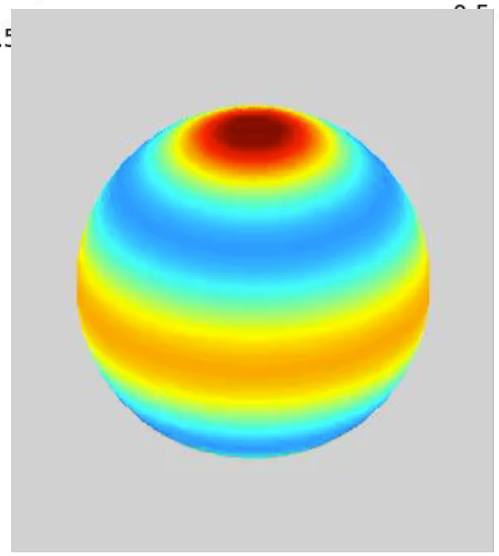


$$\frac{\partial u}{\partial t} = -u + \int_{\Omega} w(\mathbf{r}|\mathbf{r}')f(u(\mathbf{r}', t - \tau(\mathbf{r}|\mathbf{r}')))d\mathbf{r}'$$





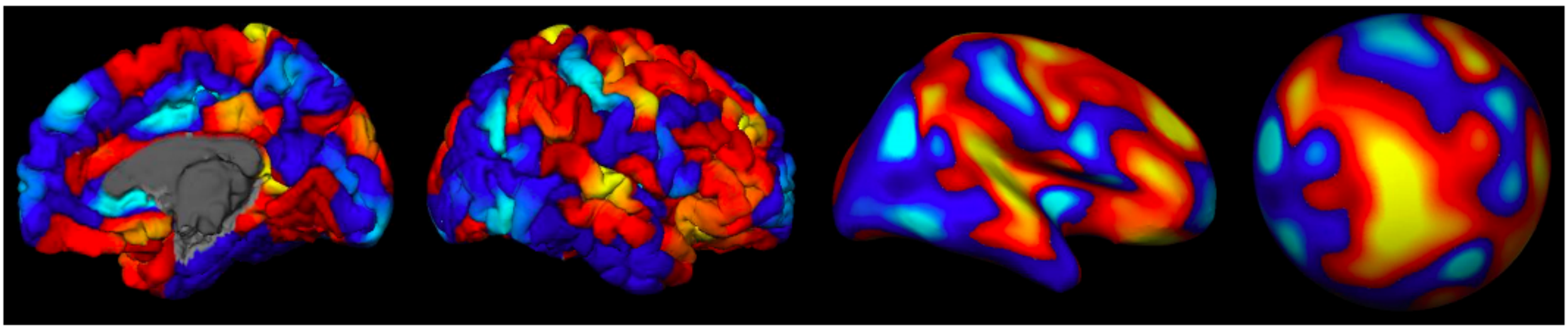




Symmetric bifurcation theory

Normal form analysis

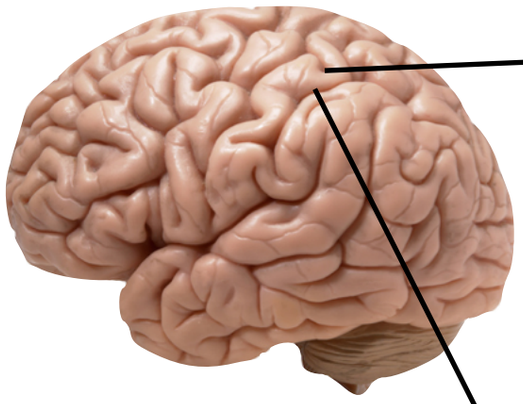
synchrony assumed / cannot rebound

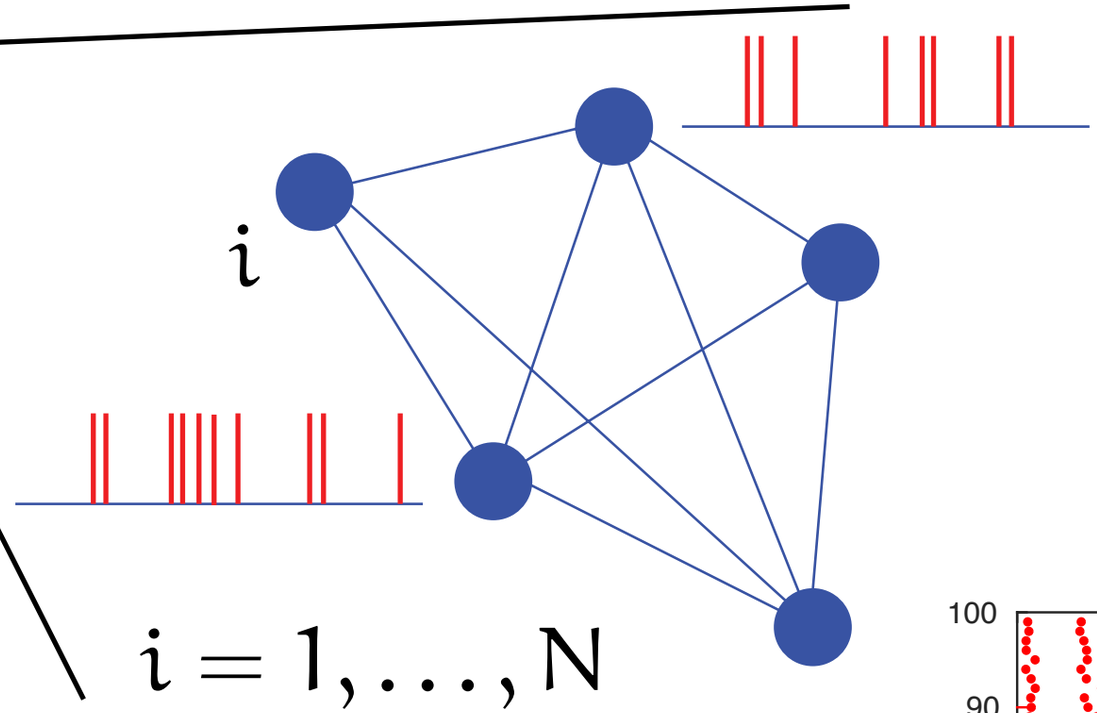


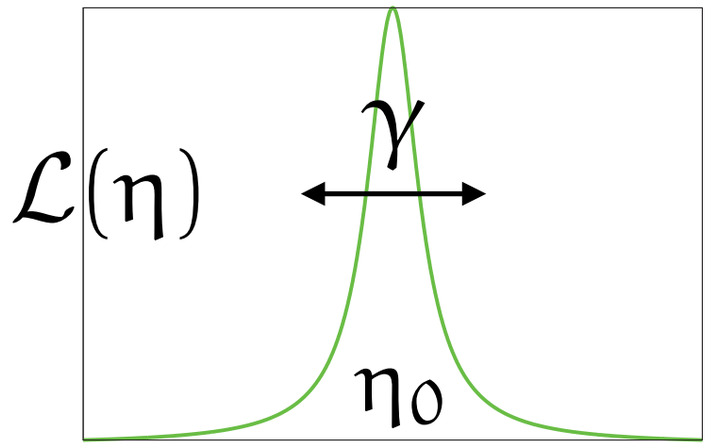
VK. Jirsa, *et al.* Spatiotemporal Forward Solution of the EEG and MEG Using Network Modeling, IEEE Trans. Med. Img., Vol. 21, 2002

An *exact* mean field model

Ask Carlo!

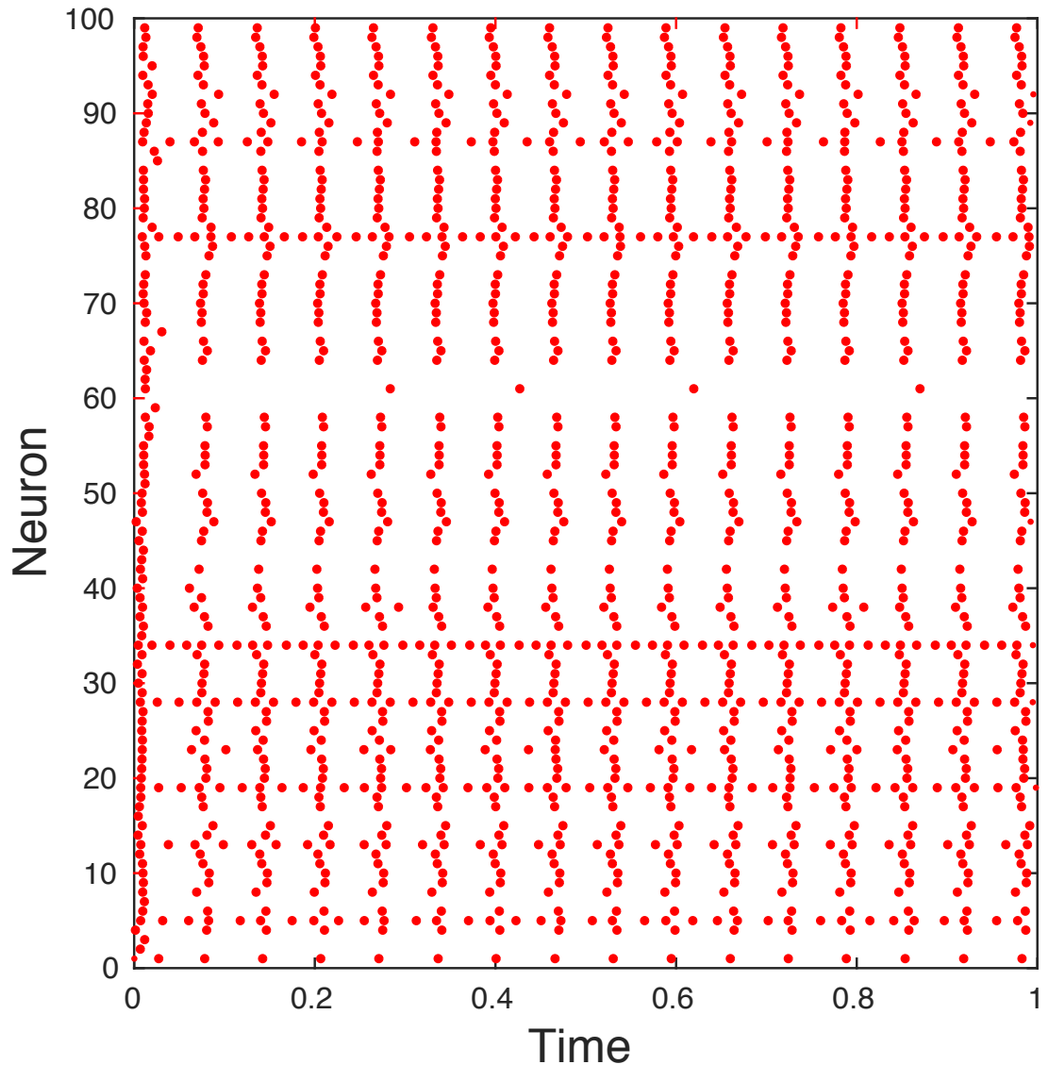


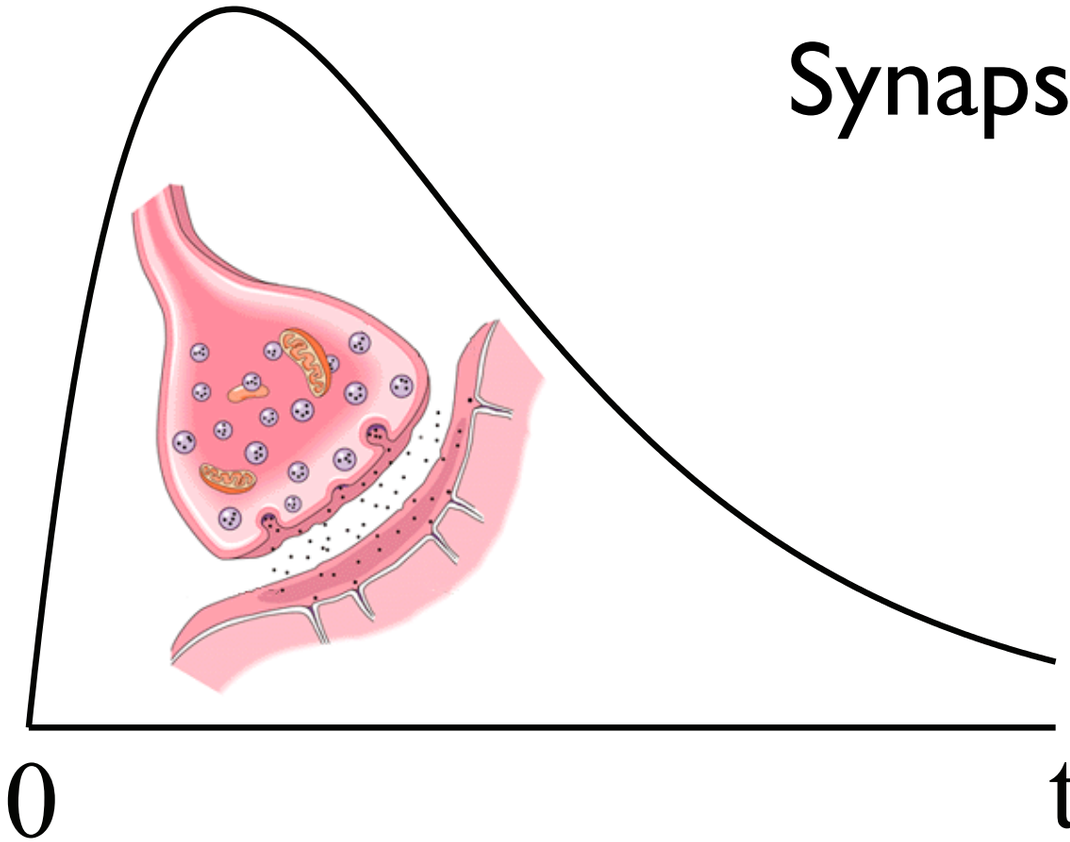




$$\frac{d}{dt}v_i = \eta_i + v_i^2 + g(t)(v_{syn} - v_i)$$

$$Qg = \frac{w}{N}\sum_{j=1}^{N}\sum_{m}\delta(t - T_j^m)$$





Synapse model

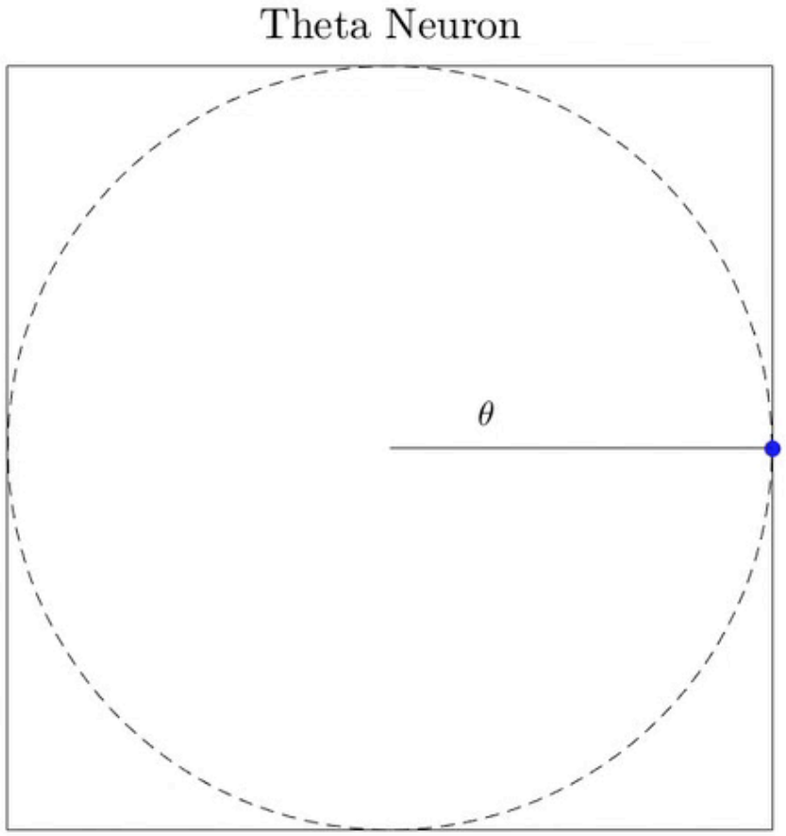
$$Q\eta = \delta$$

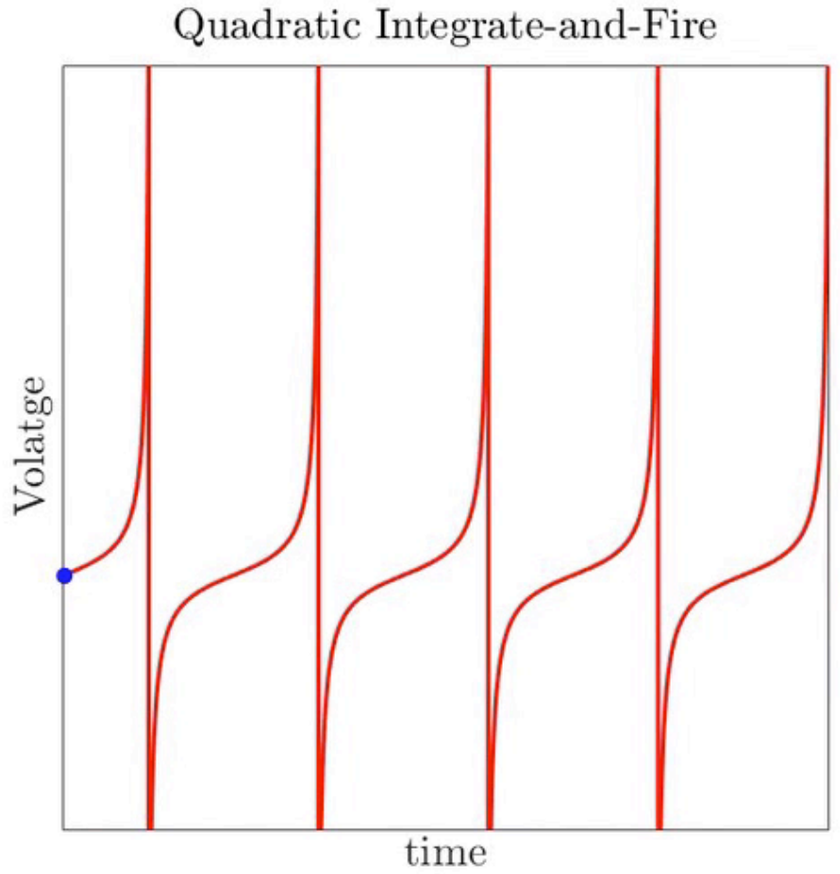
$$Q = \left(1 + \frac{1}{\alpha}\frac{d}{dt}\right)^2$$

$$\eta(t) = \alpha^2 t e^{-\alpha t}$$

Phase variable (theta neuron)

$$v_i = \tan(\theta_i/2)$$





Routes to mean field $\mathrm{N} \to \infty$

Continuity eqn for density $\rho_t + \nabla \cdot J = 0$

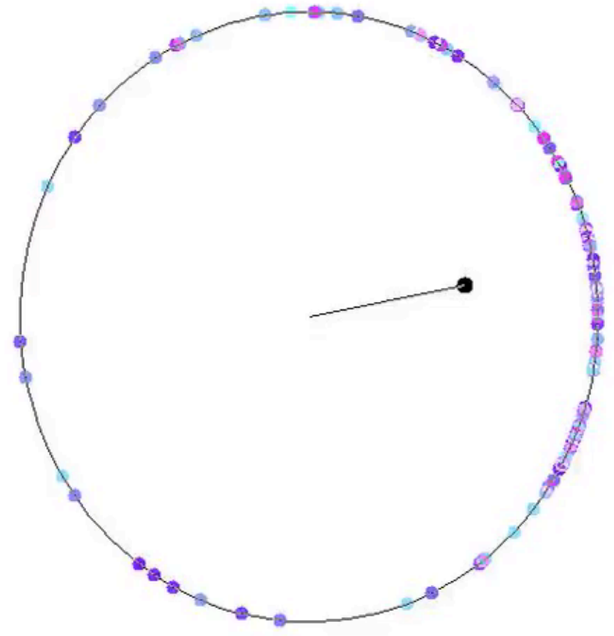
TB Luke, E Barreto, P So 2013 (theta)

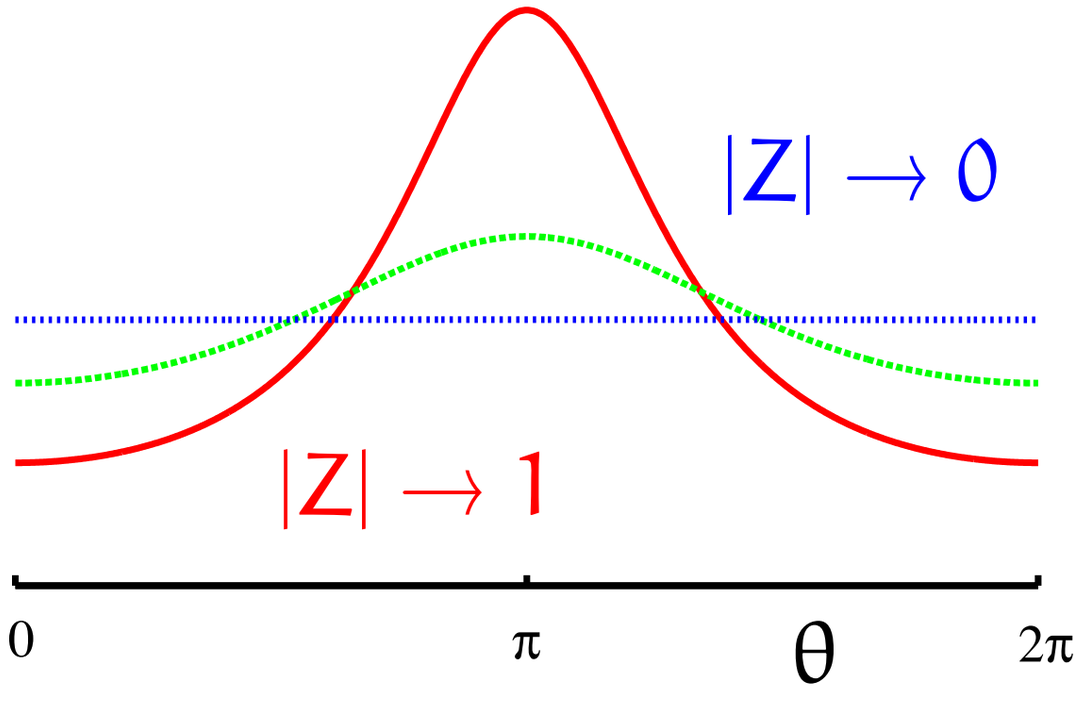
$$\rho(\theta \mid \eta, t)/\mathcal{L}(\eta)$$

$$\frac{1}{2\pi} \sum_n a^n(\eta, t) e^{in\theta}$$

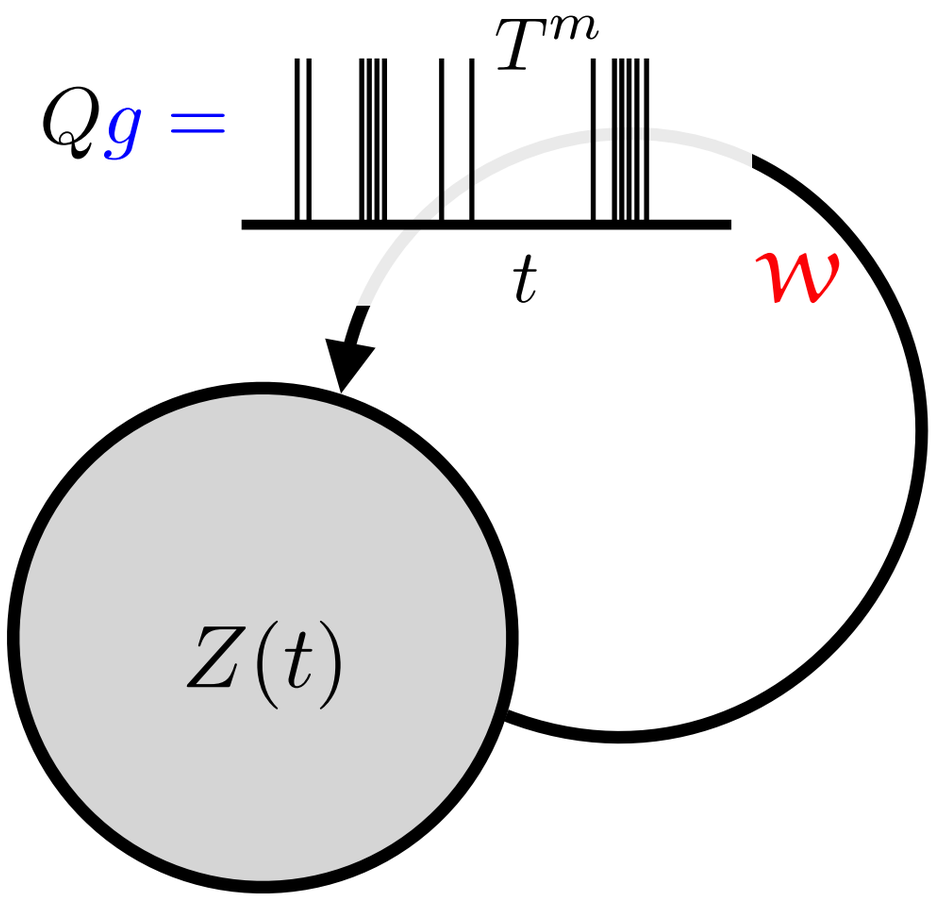
OA ansatz

$$Z = \langle a^* \rangle_\eta = \langle e^{i\theta} \rangle$$



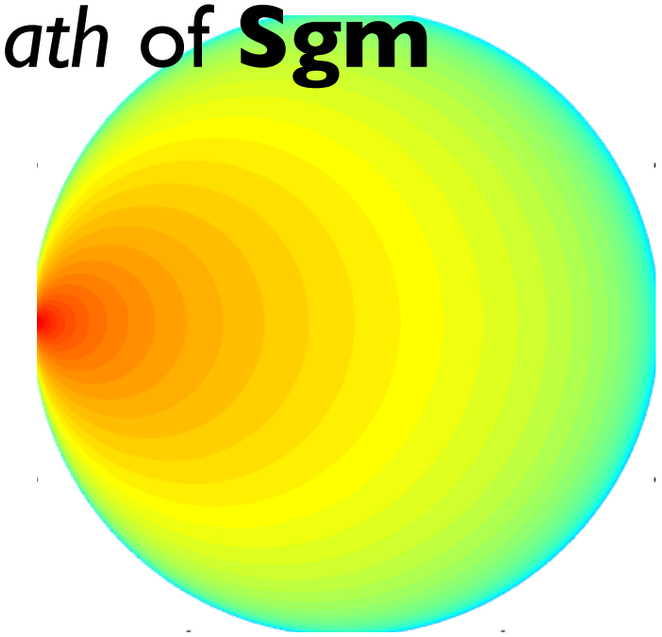


Dynamics are *neural mass* like $\mathrm{Q}g = w\mathrm{R}$



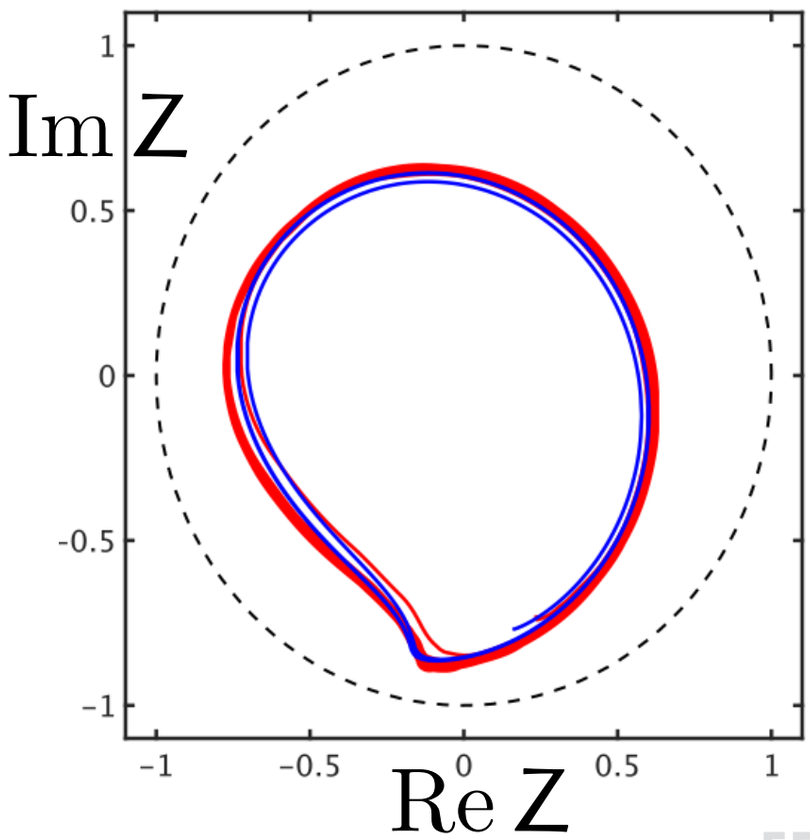
$$\mathrm{R} = \frac{1}{\pi}\mathrm{Re}\left(\frac{1-\mathrm{Z}^*}{1+\mathrm{Z}^*}\right)$$

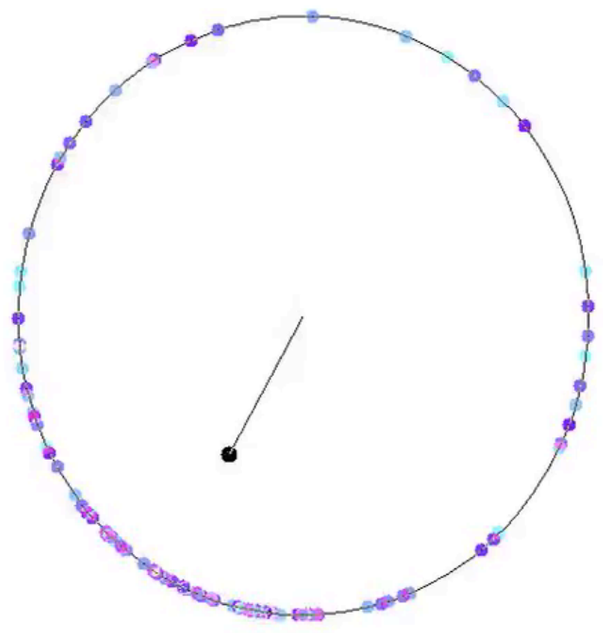
Death of **Sgm**



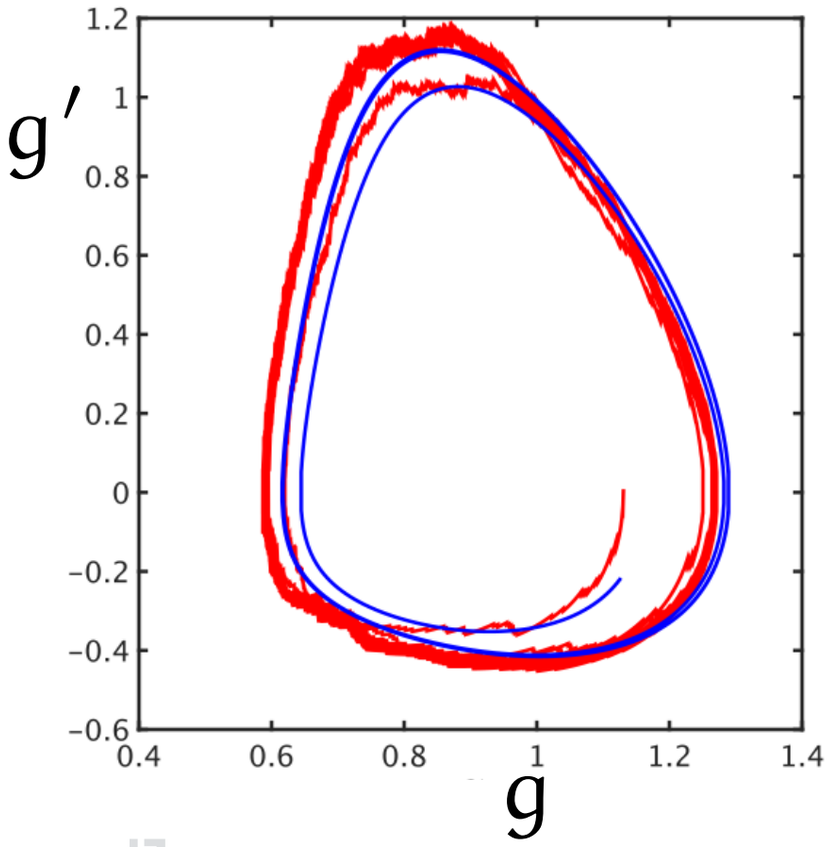
$$\frac{\mathrm{d}}{\mathrm{dt}}\mathrm{Z} = -\mathrm{i}\frac{(\mathrm{Z}-1)^2}{2} + \frac{(\mathrm{Z}+1)^2}{2}[-\gamma + \mathrm{i}\eta_o + \mathrm{i}v_{\mathrm{syn}}\mathrm{g}] - \frac{(\mathrm{Z}^2-1)}{2}\mathrm{g}$$

Next generation rate *and* synchrony model

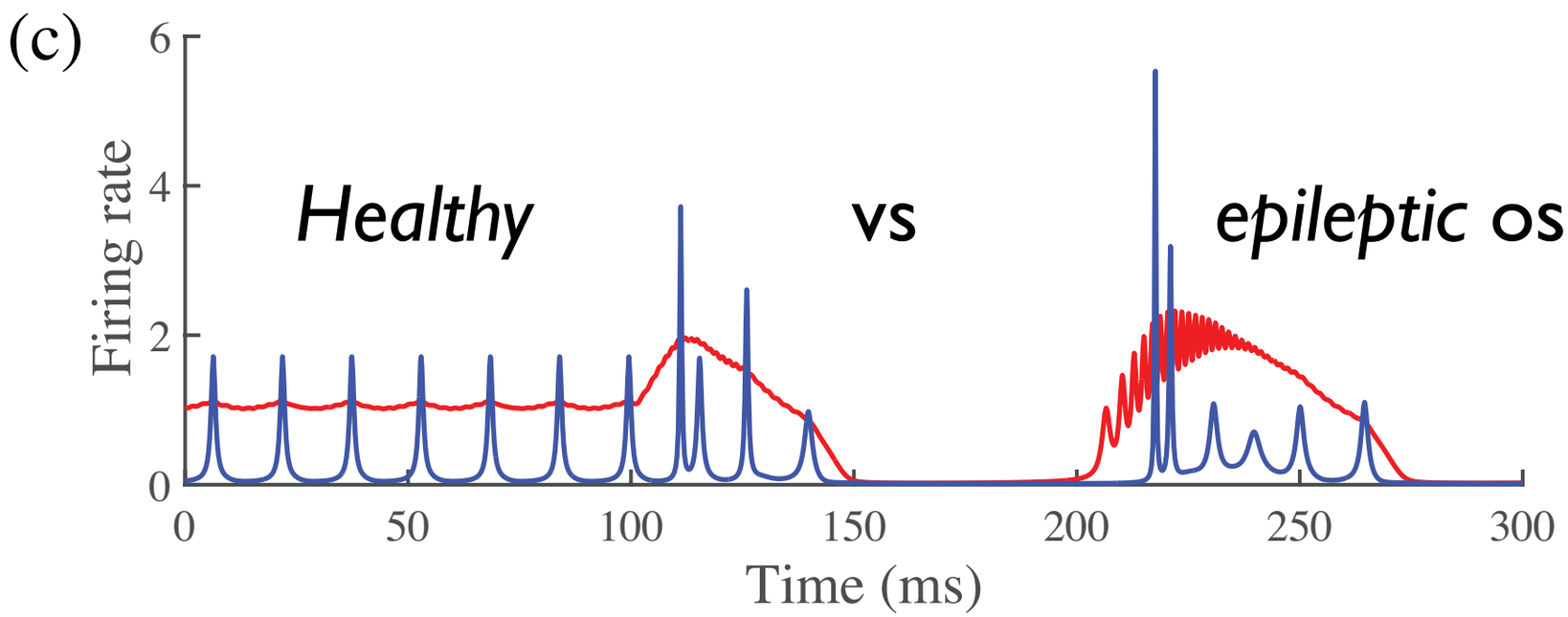




$N = 500$

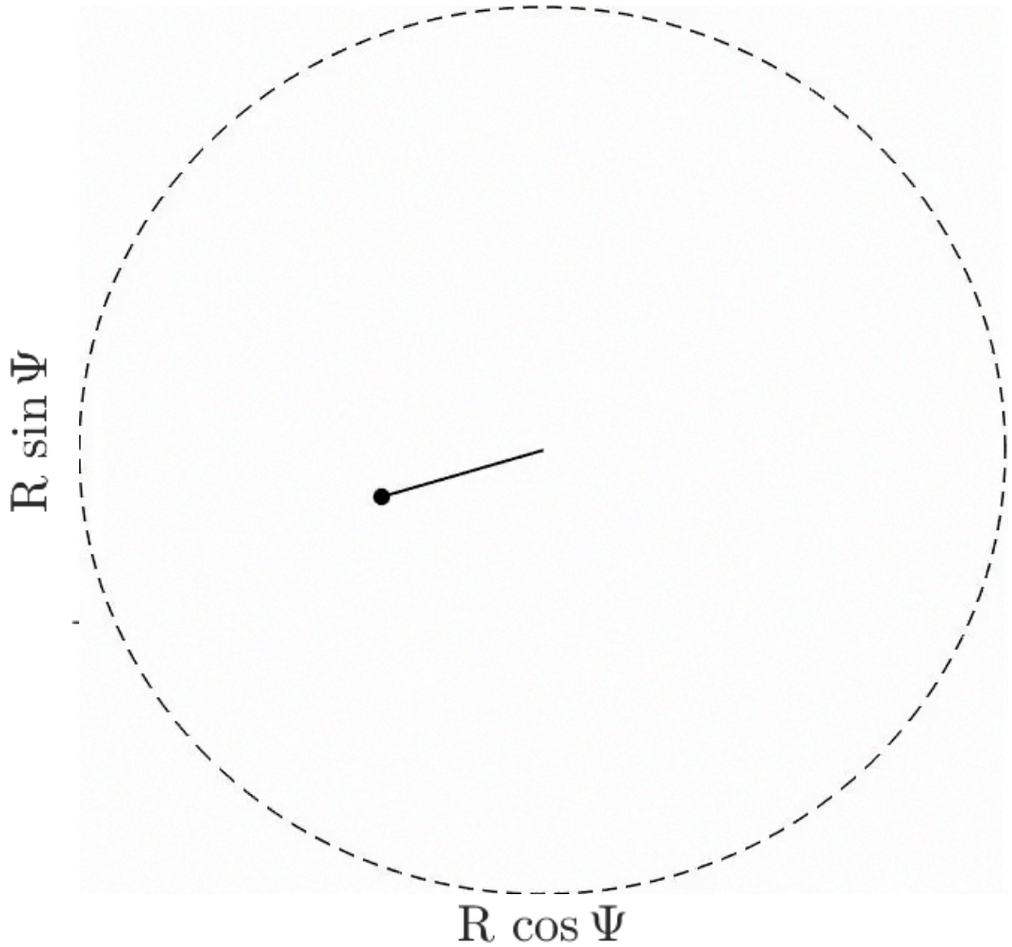


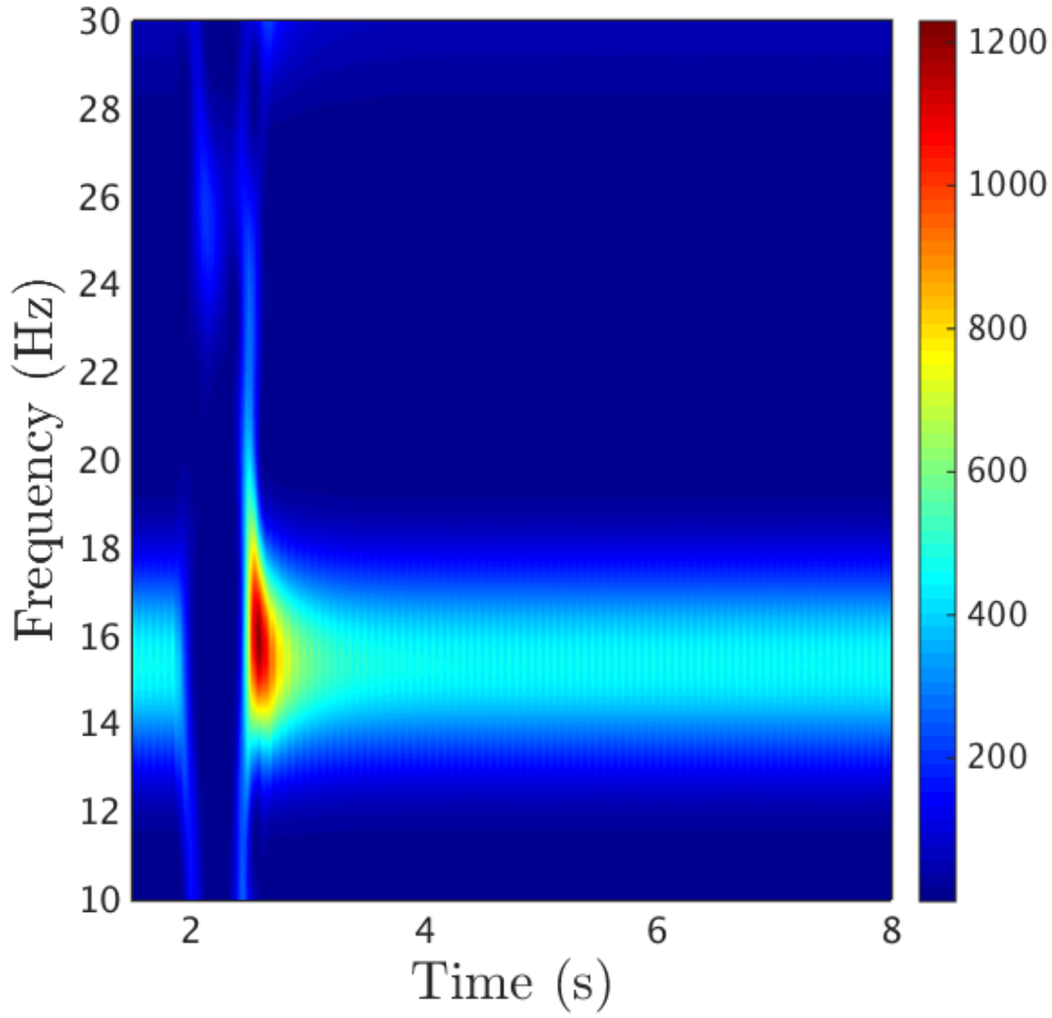
[Finite size effects expected]



Self inhibition and a excitatory background drive

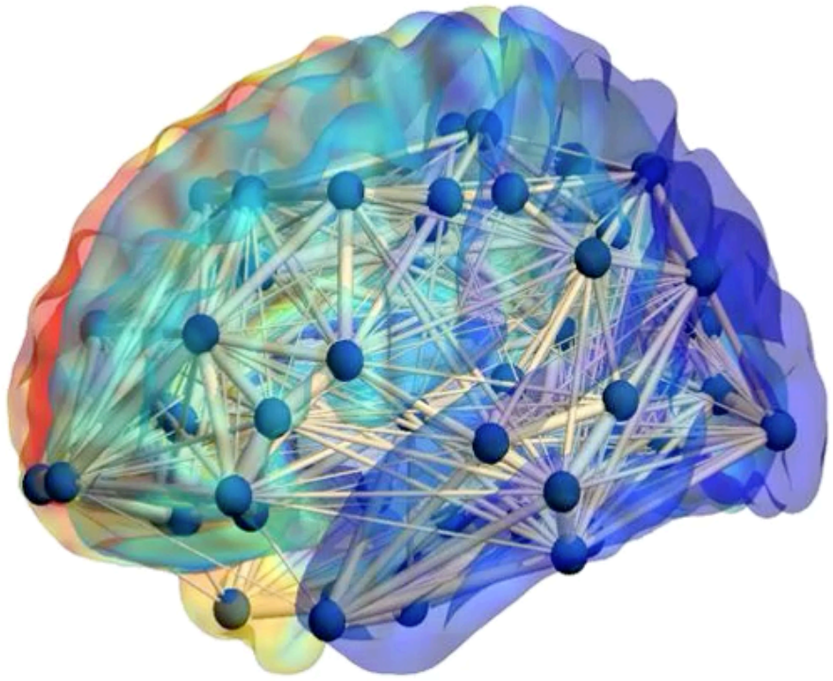
(square pulse response)

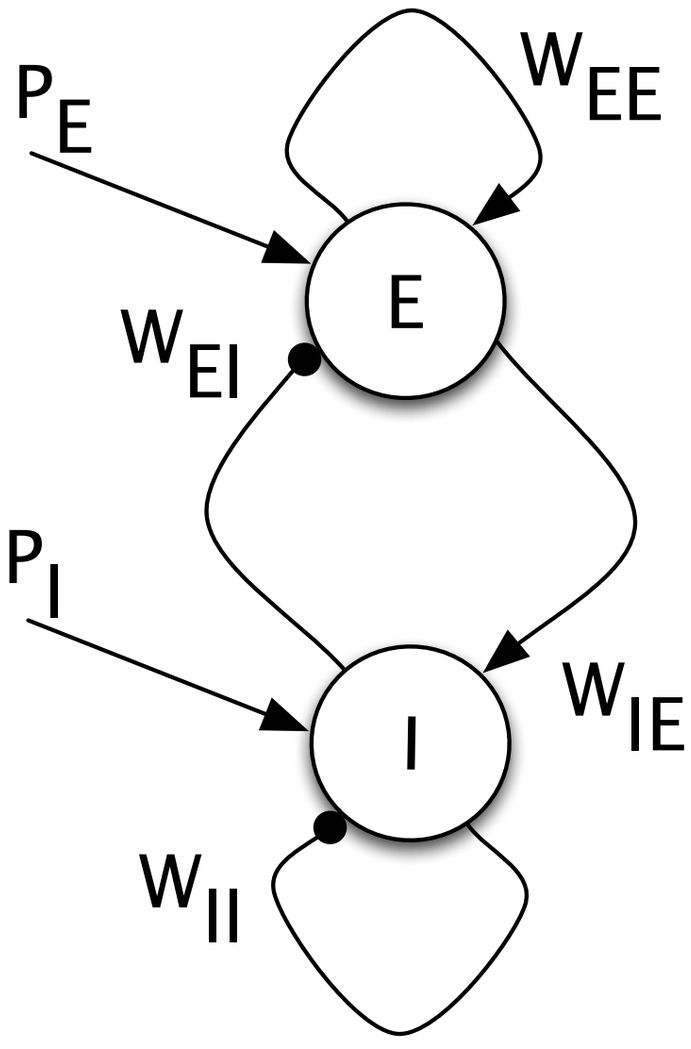




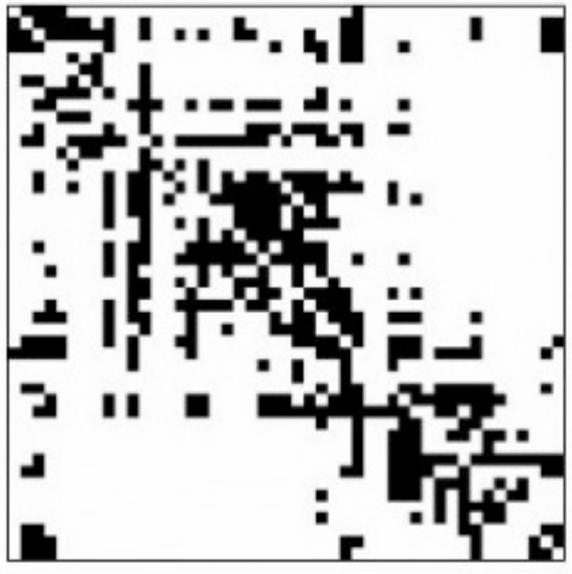
A Byrne, M Brookes and S Coombes 2017
A mean field model for movement induced changes in the β rhythm, JCNS, Vol 43, 143-158

Networks





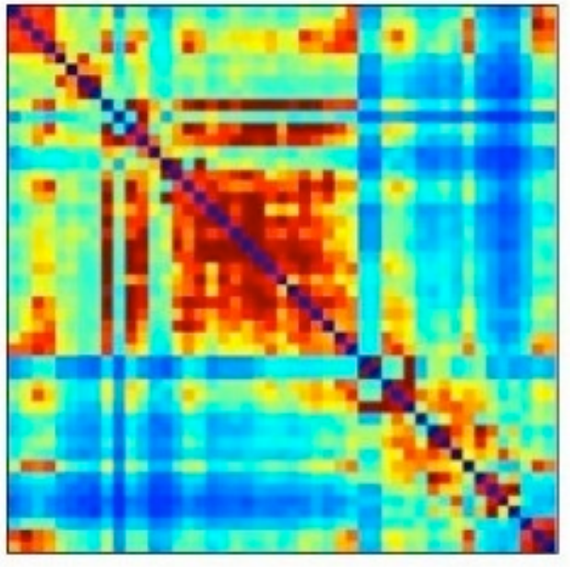
+



Structural connectivity

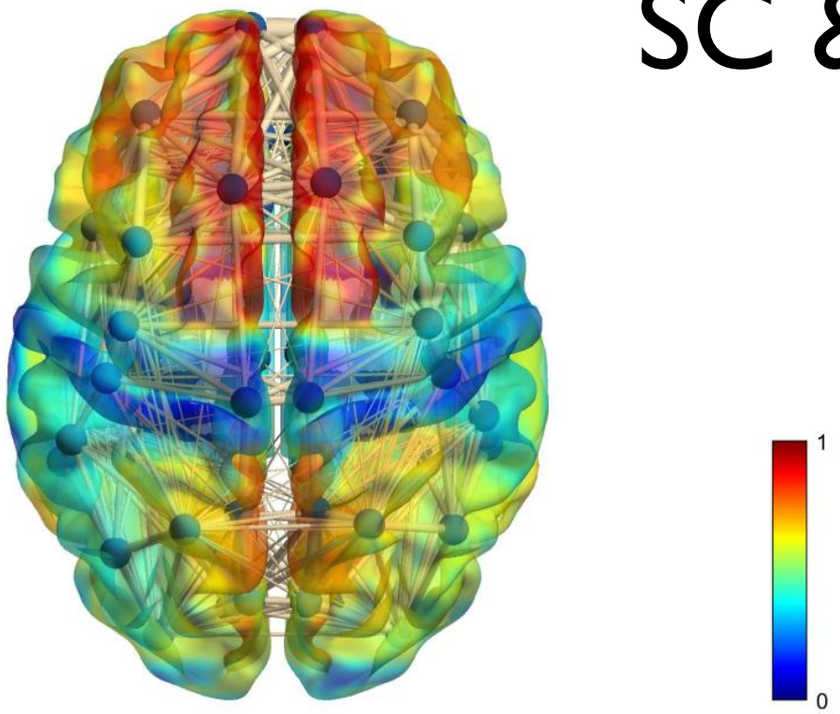
?

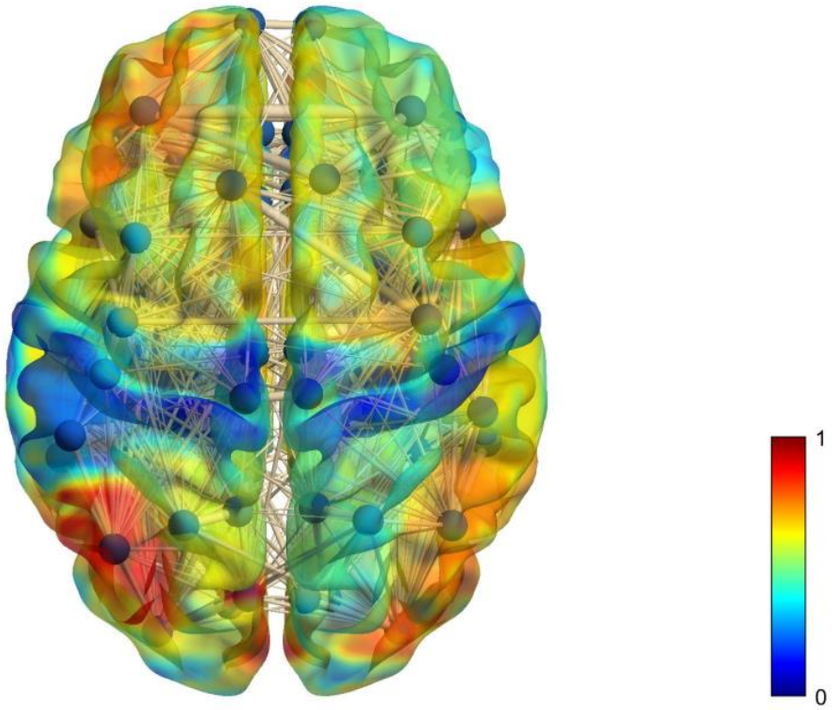
=



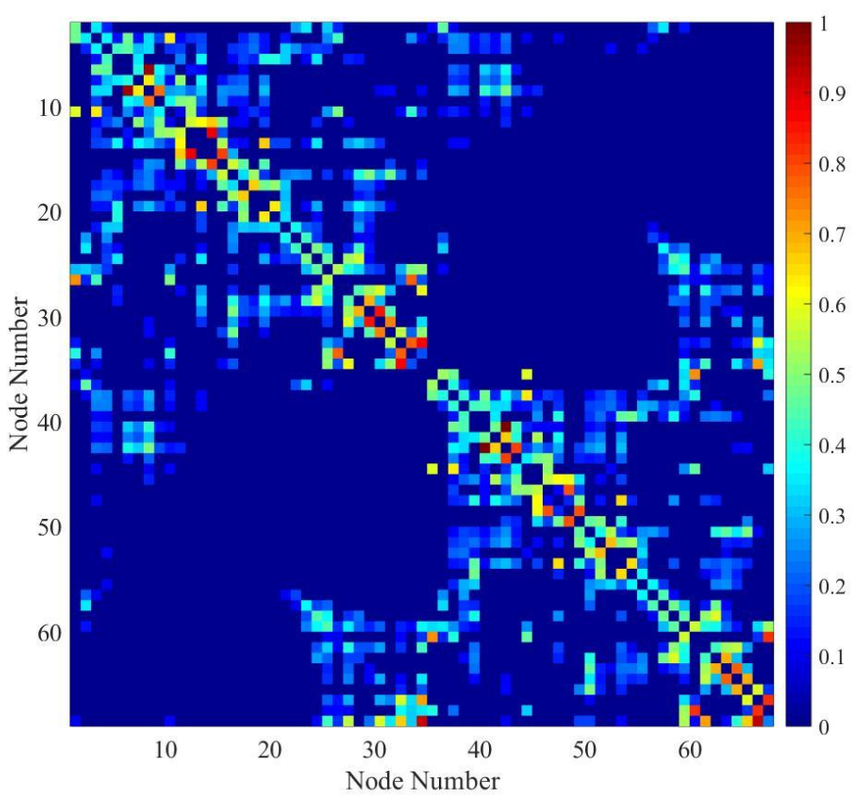
Functional connectivity

SC & FC

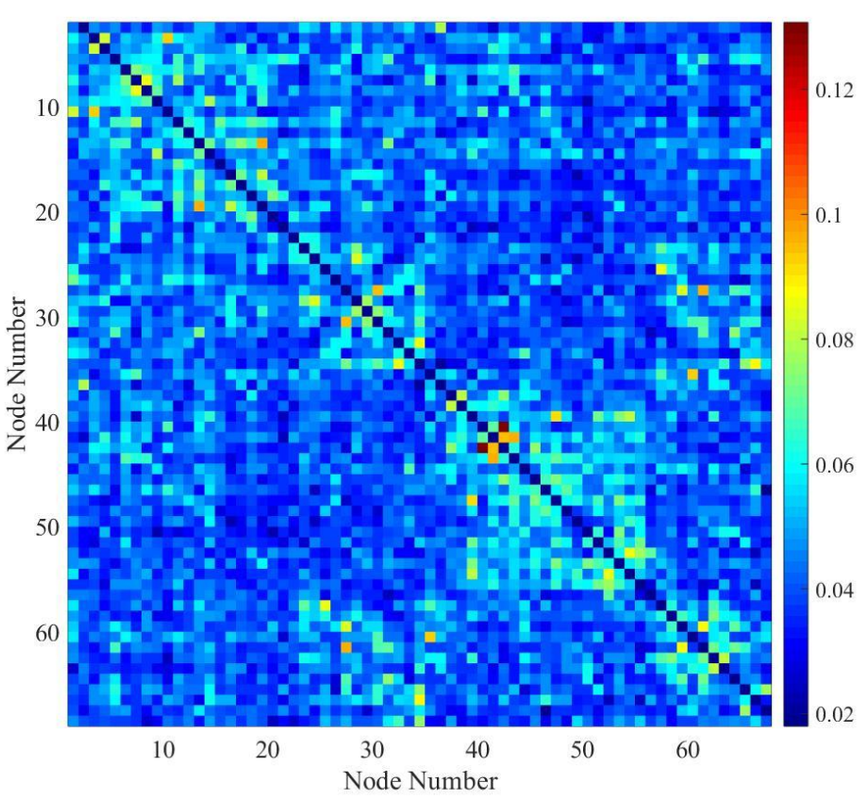




(a)

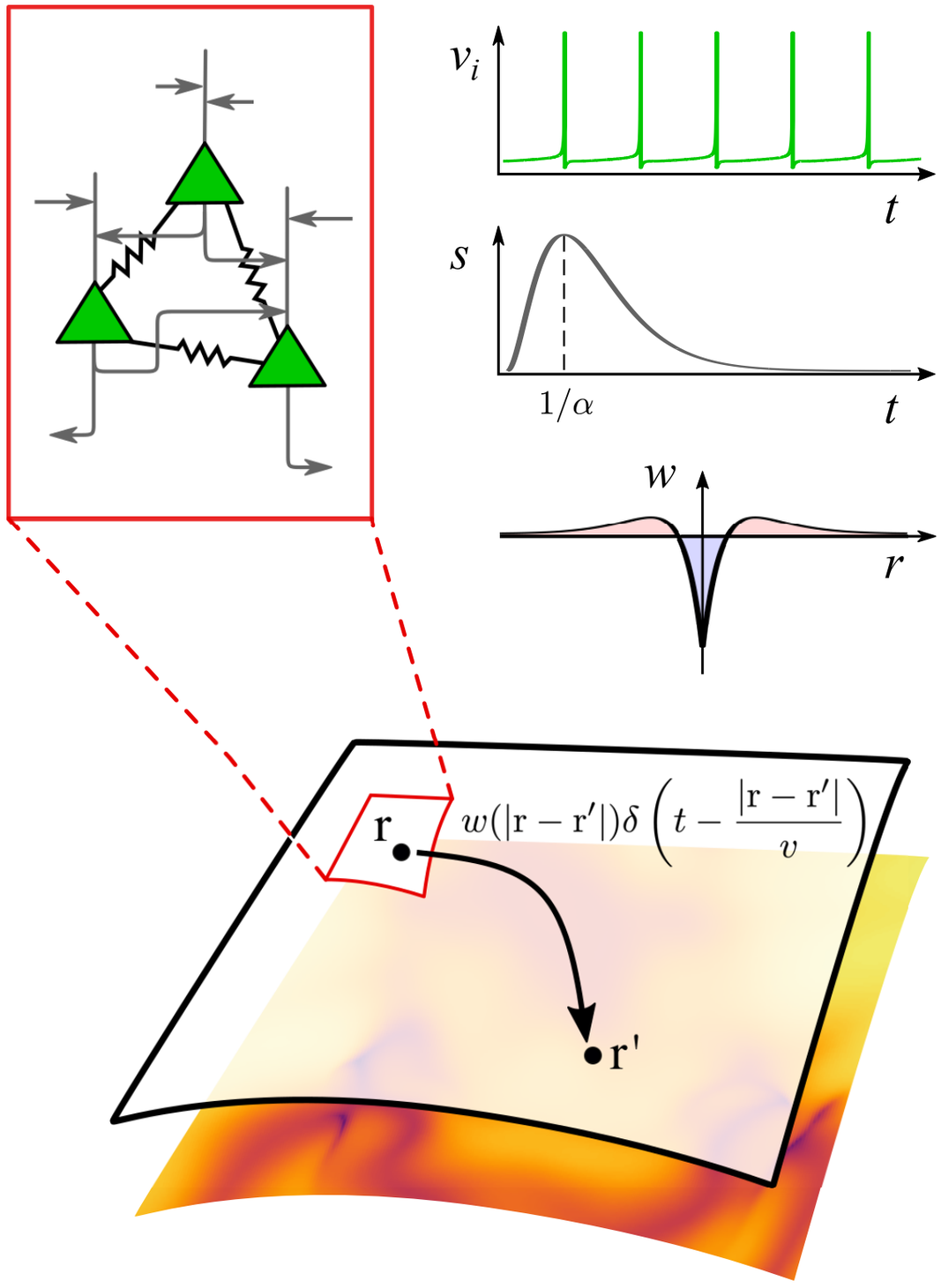


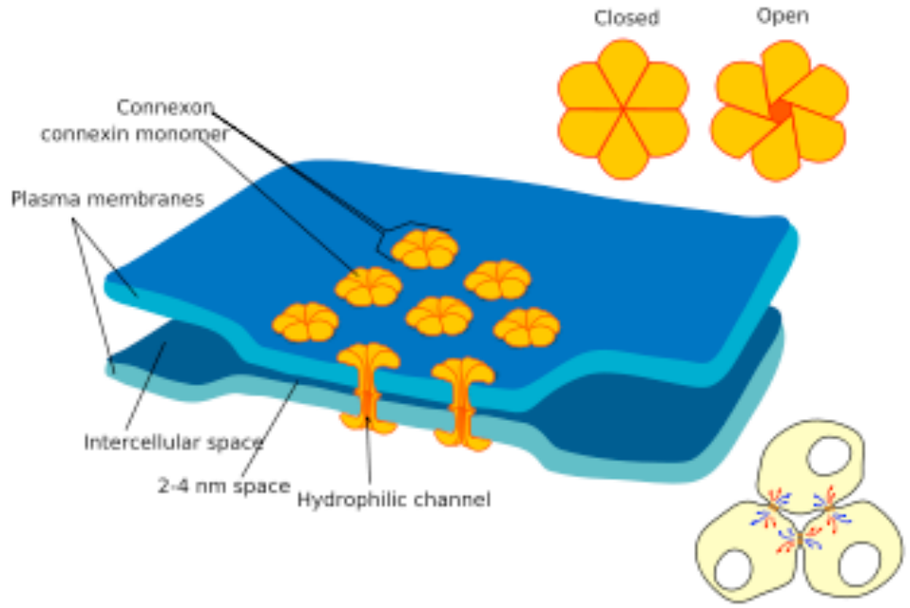
(b)

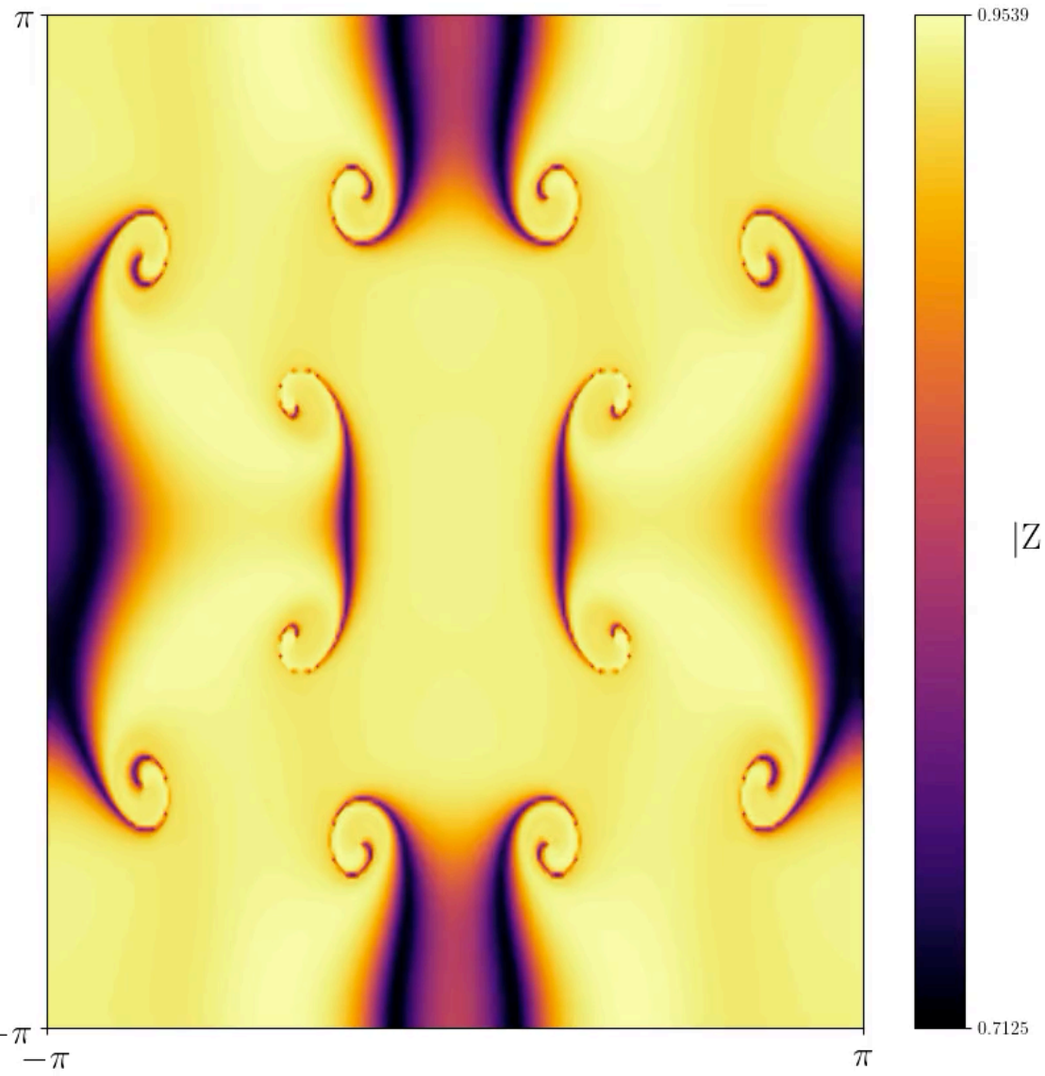


[surface of the brain visualisations coloured depending on nodal degree]

Sheets: including gap junctions & axonal delays



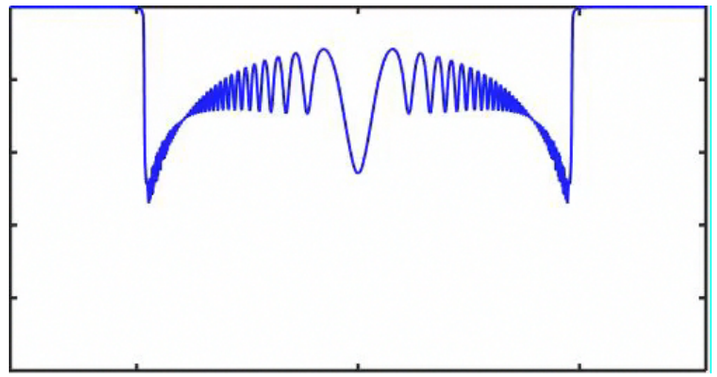




Turing instability analysis

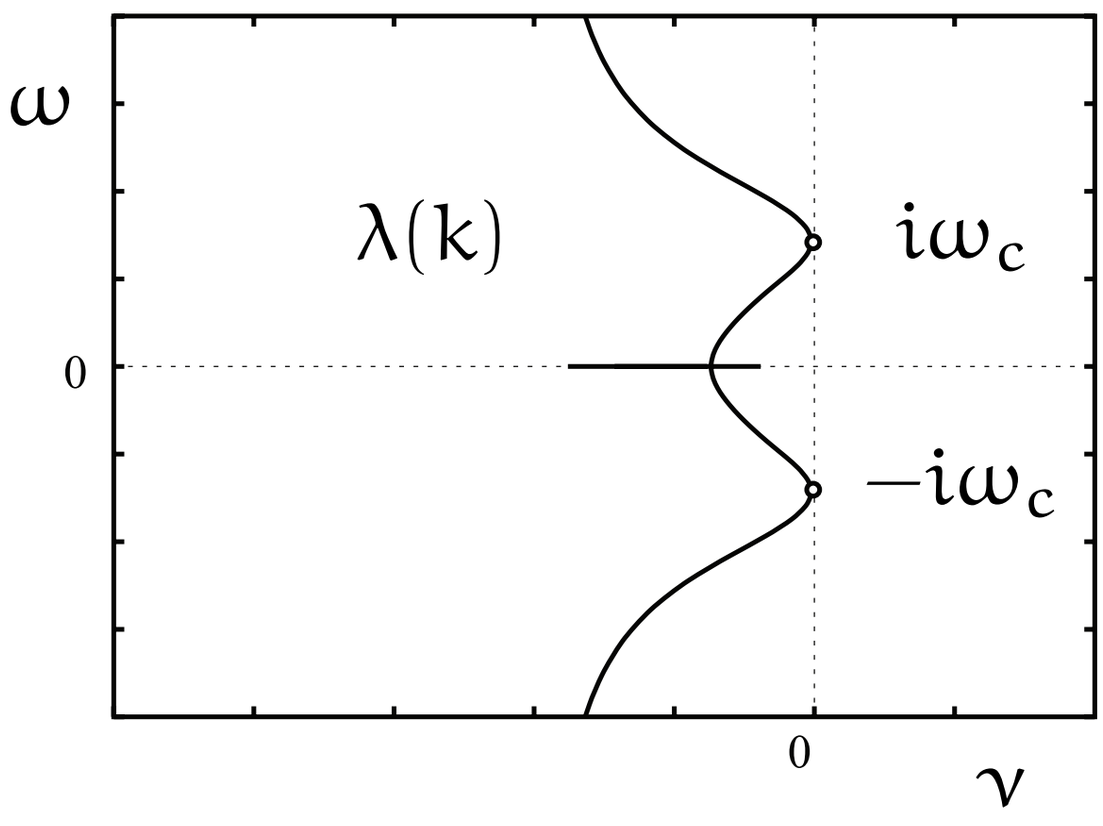
Linearise around a homogeneous steady state and look for patterns of the form

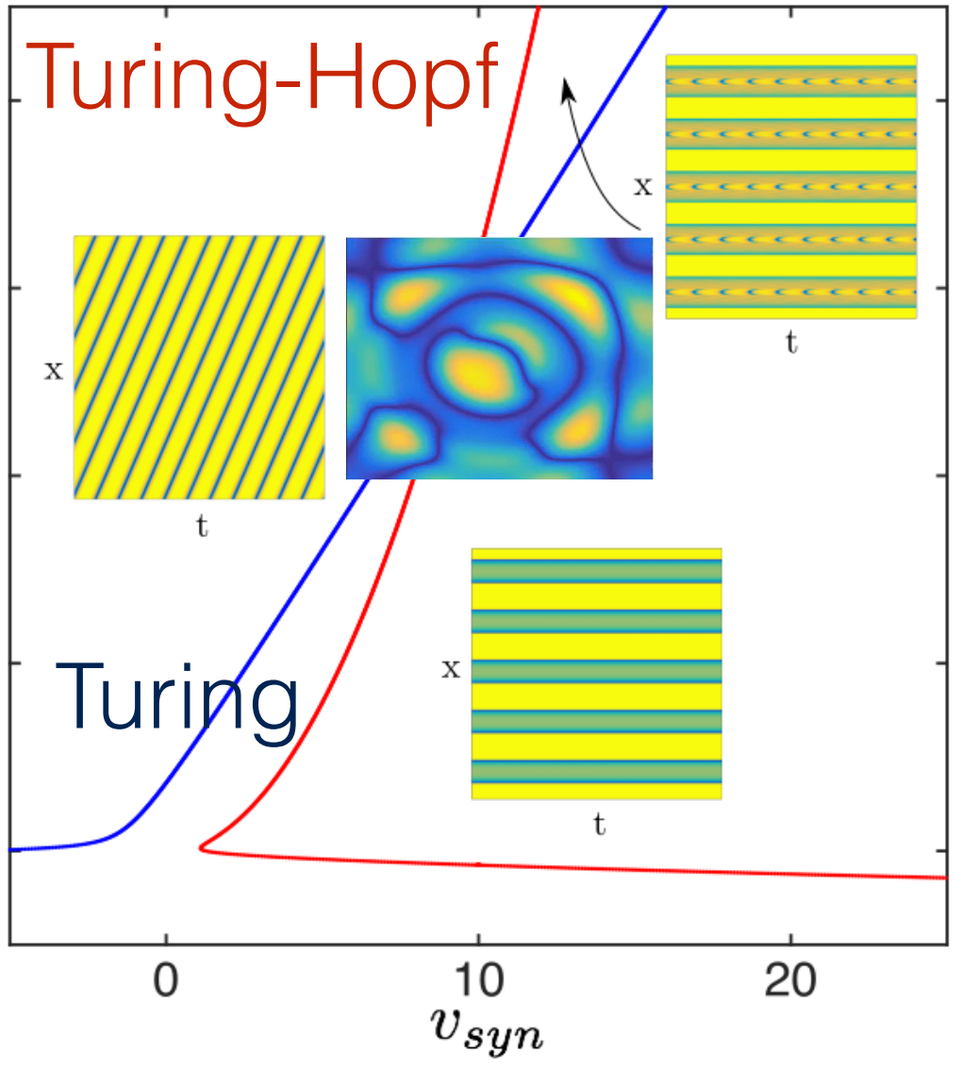
$$e^{i\mathbf{k}\cdot\mathbf{r}}e^{\lambda t}$$



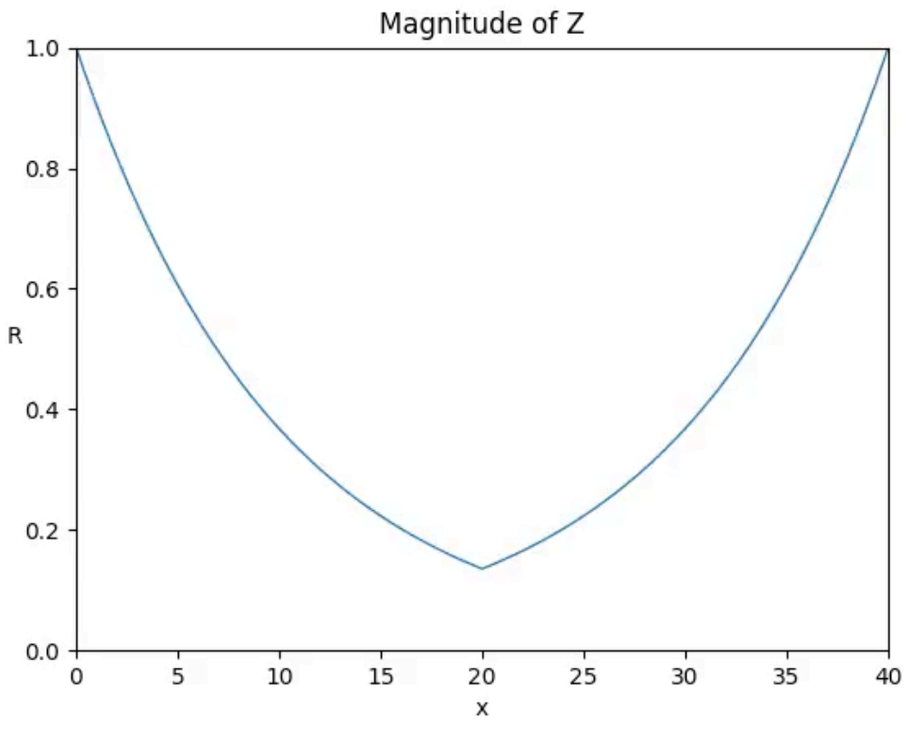
Continuous spectrum

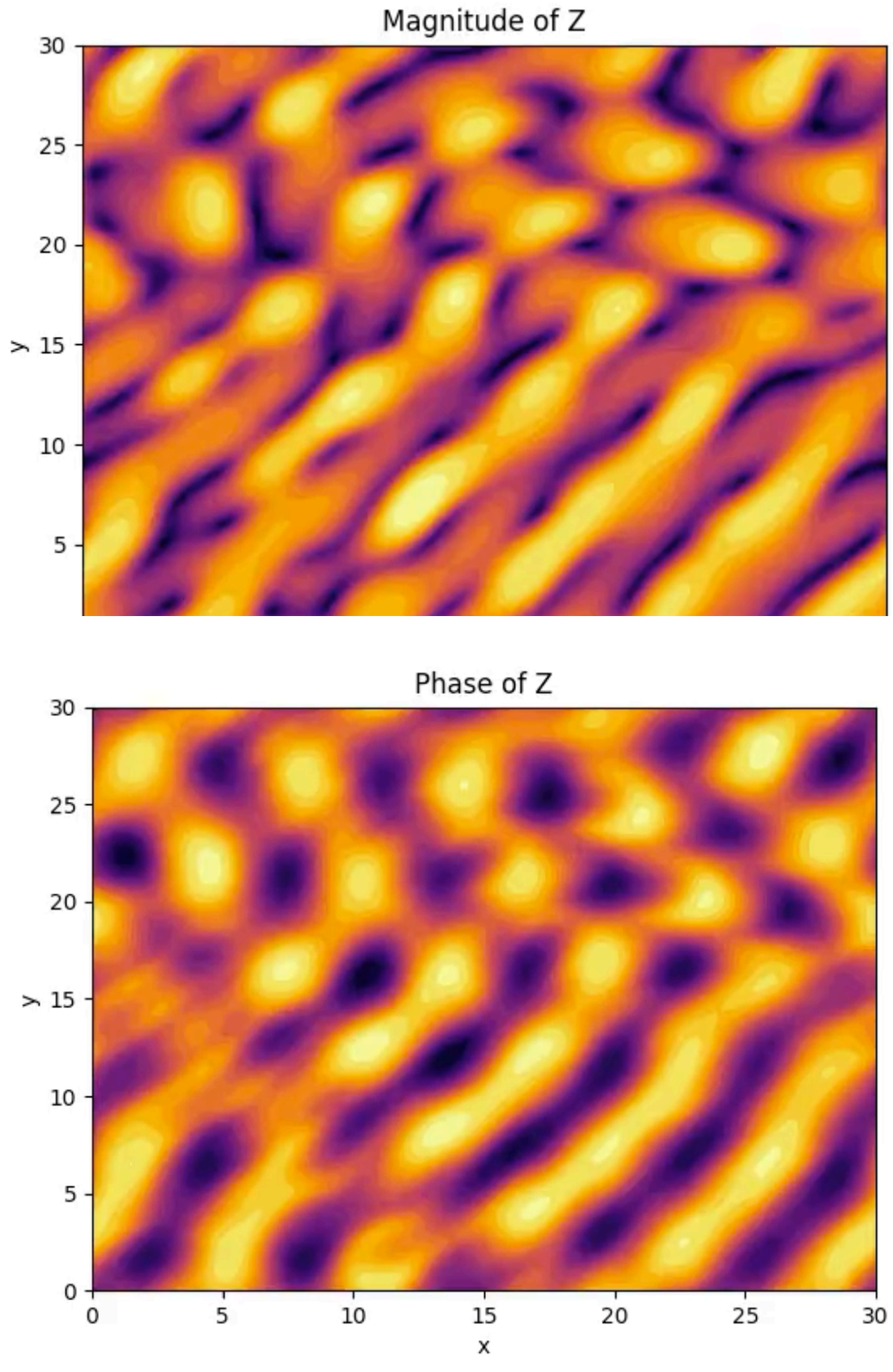
$$\lambda = \nu + i\omega$$

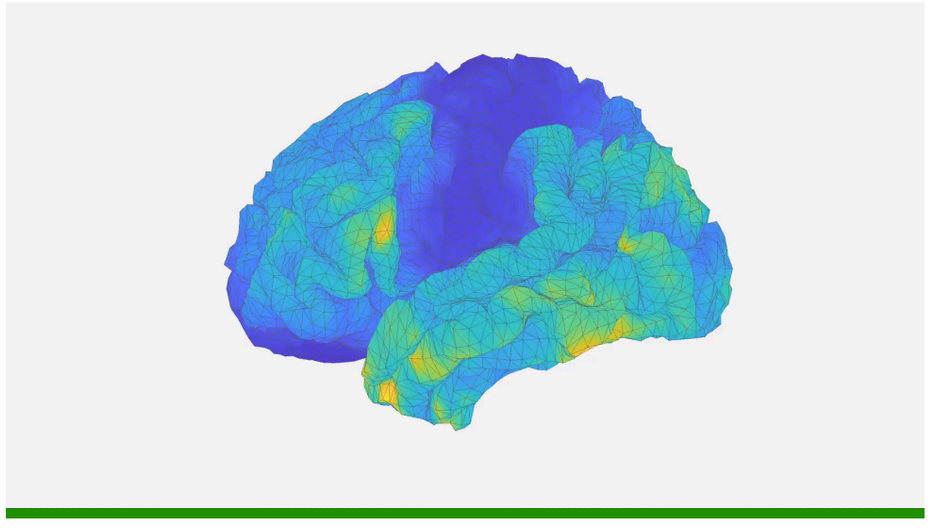




Higher dimensional sims

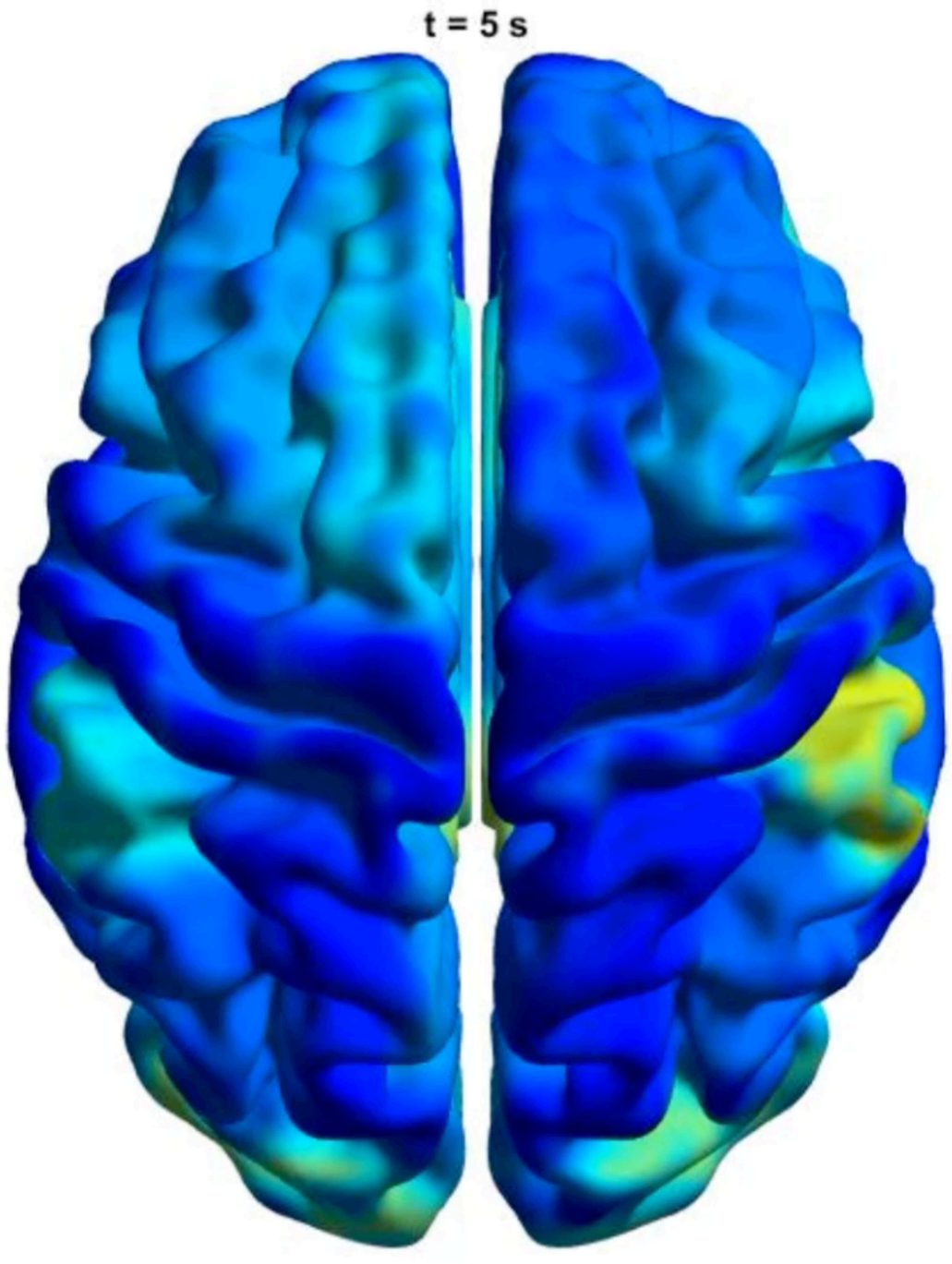


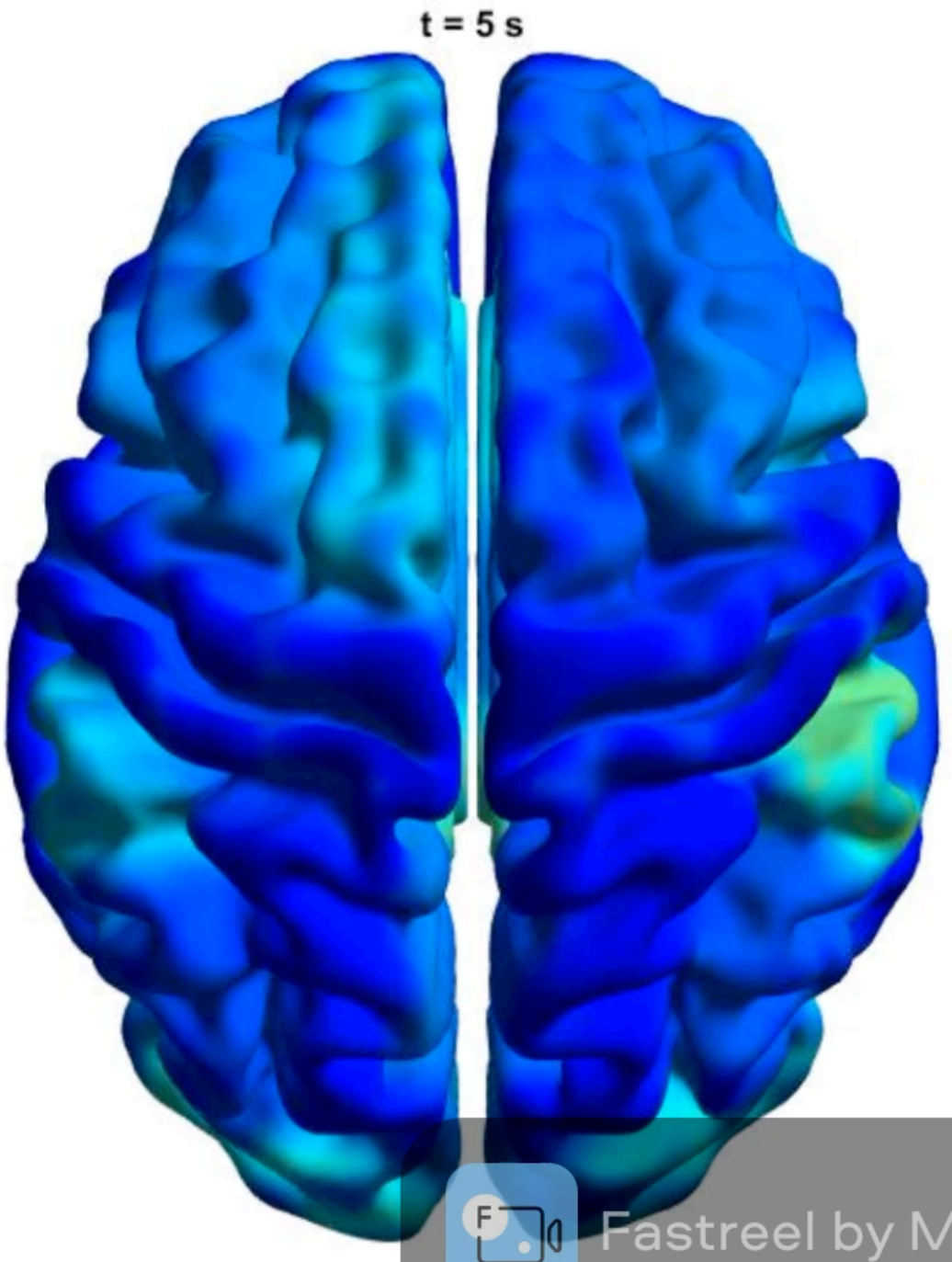




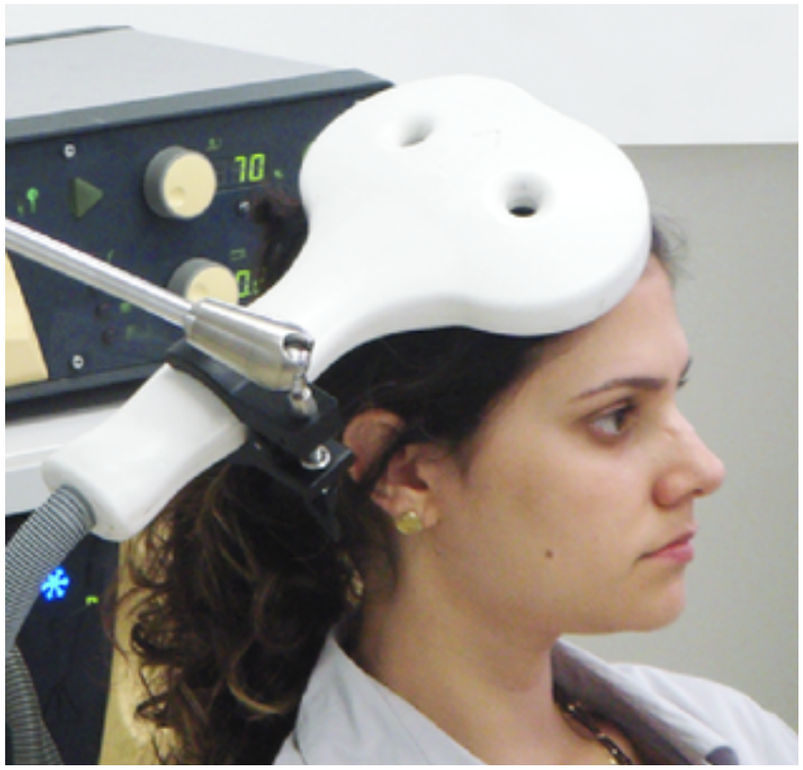
Dynamic FC (local control ~ gaps)

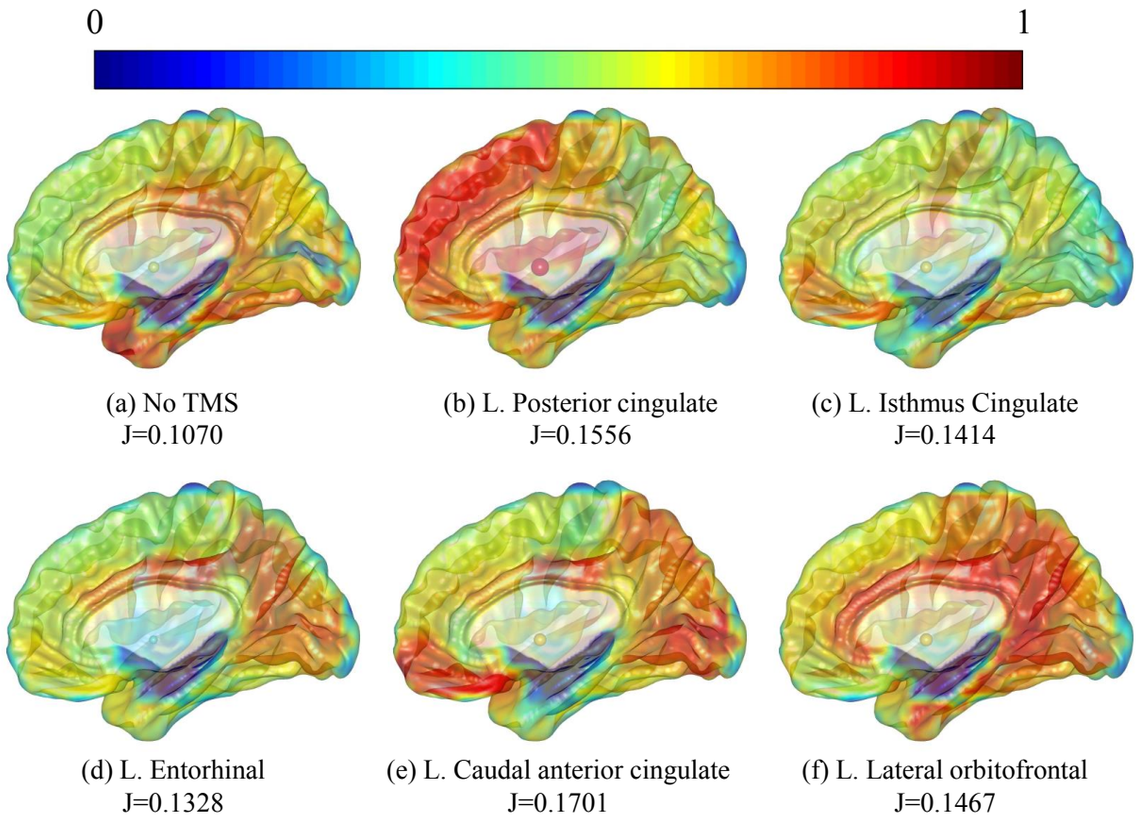
https://github.com/UoN-Math-Neuro/NFESOLVE





Exploring (r)TMS





(a) No TMS
J=0.1070

(b) L. Posterior cingulate
J=0.1556

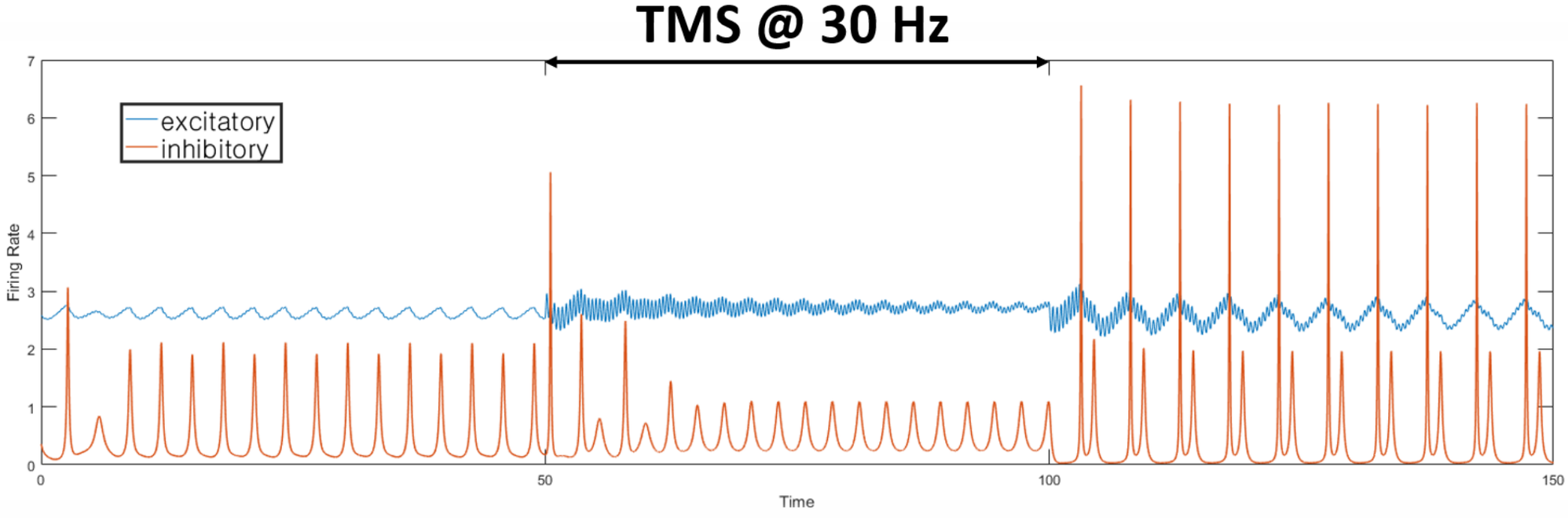
(c) L. Isthmus Cingulate
J=0.1414

(d) L. Entorhinal
J=0.1328

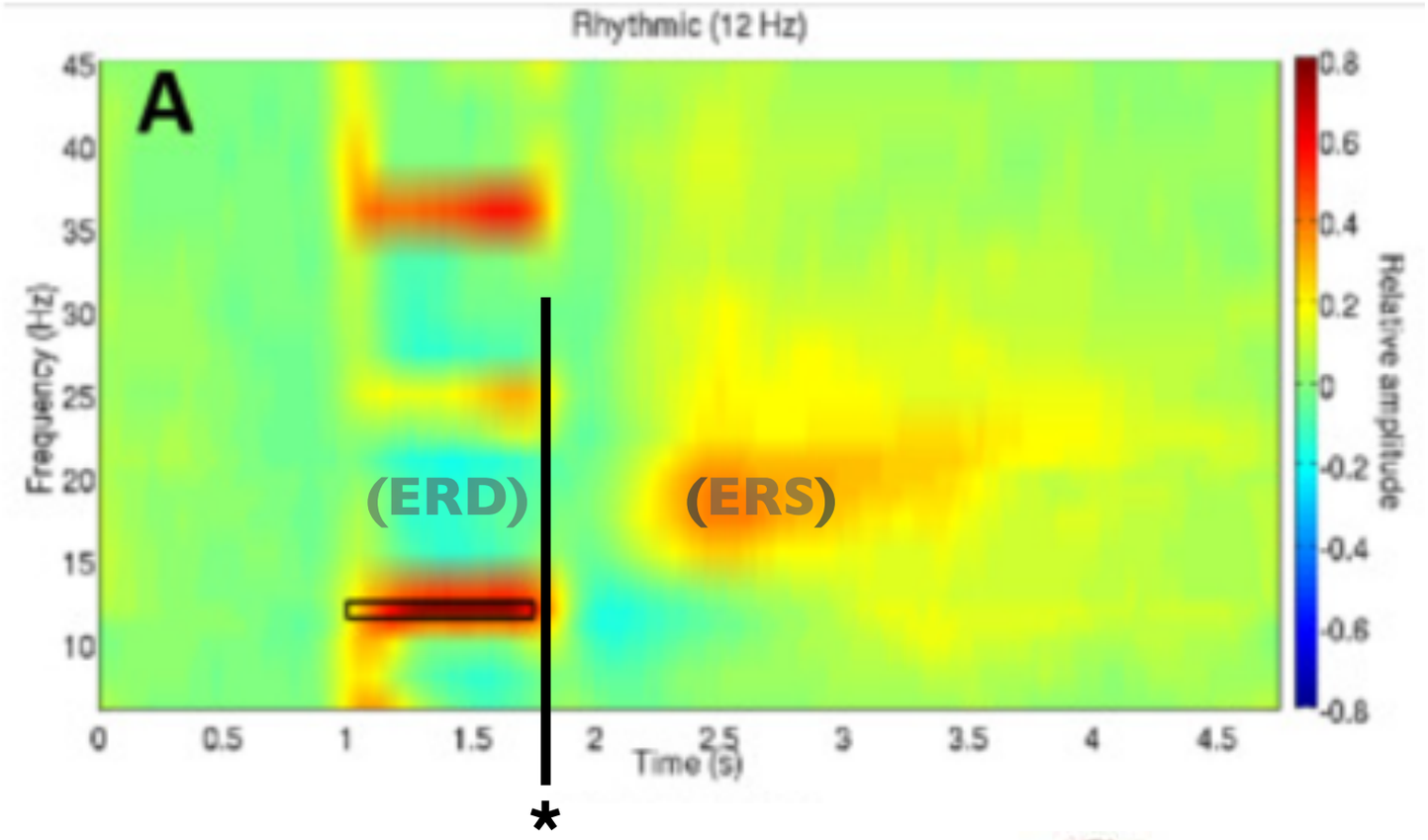
(e) L. Caudal anterior cingulate
J=0.1701

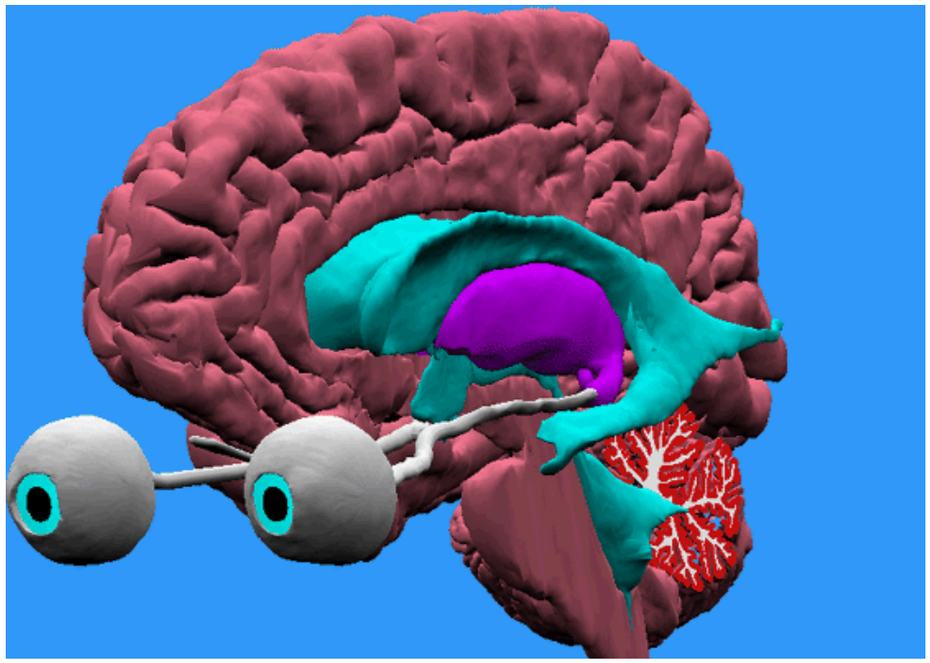
(f) L. Lateral orbitofrontal
J=0.1467

Normalised node degree of FC networks

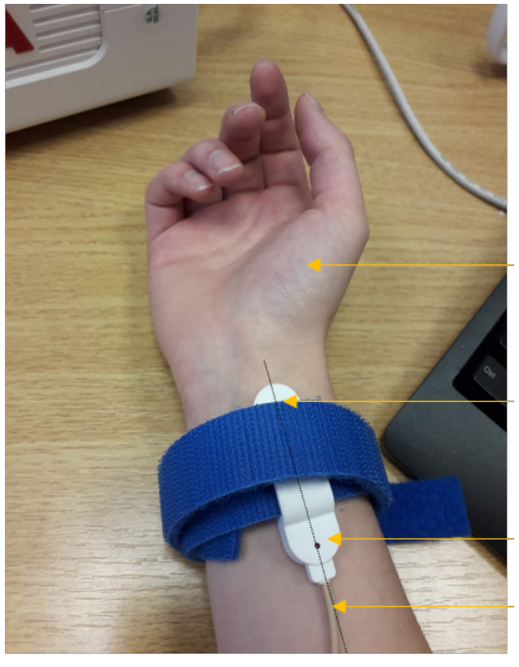


Sensory drive

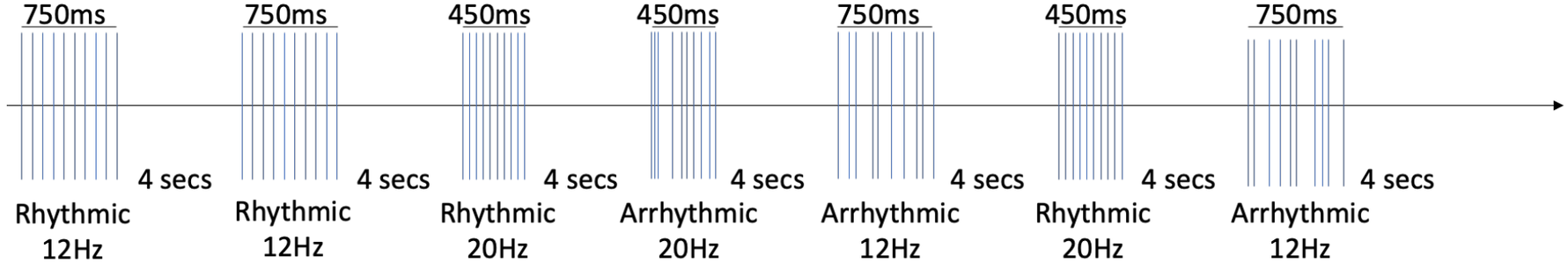




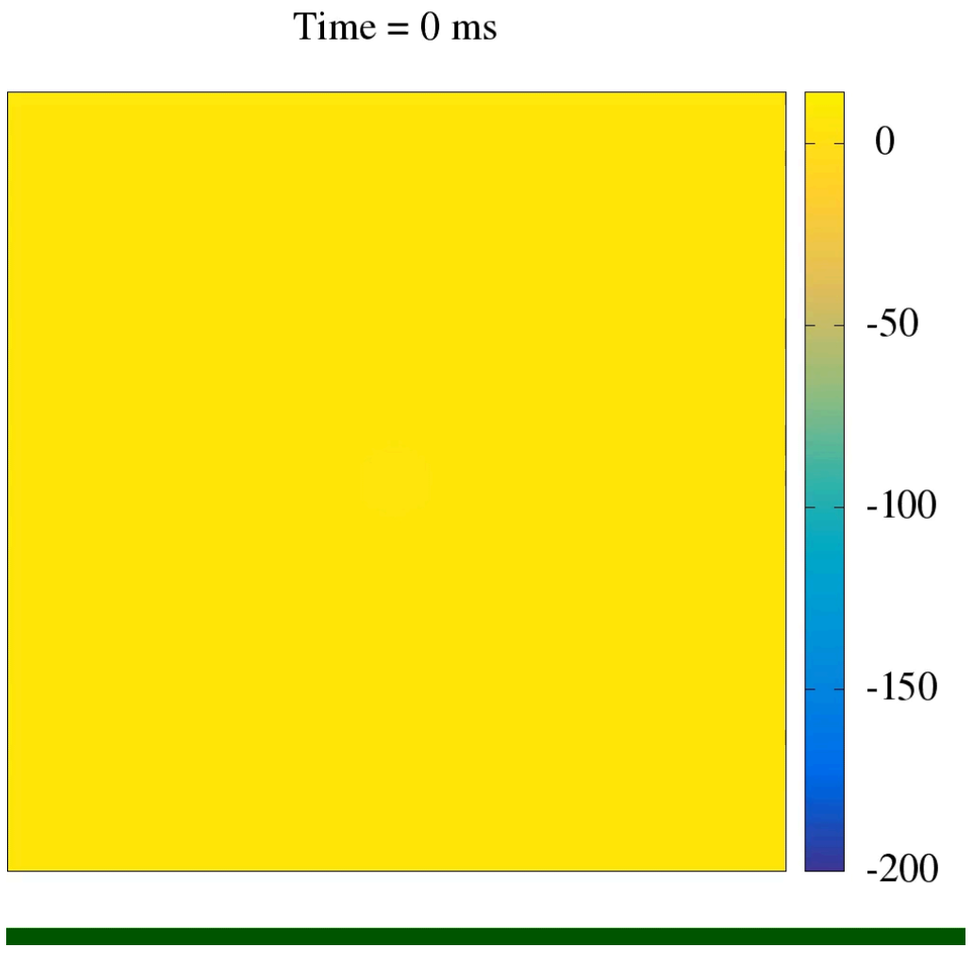
Thalamus

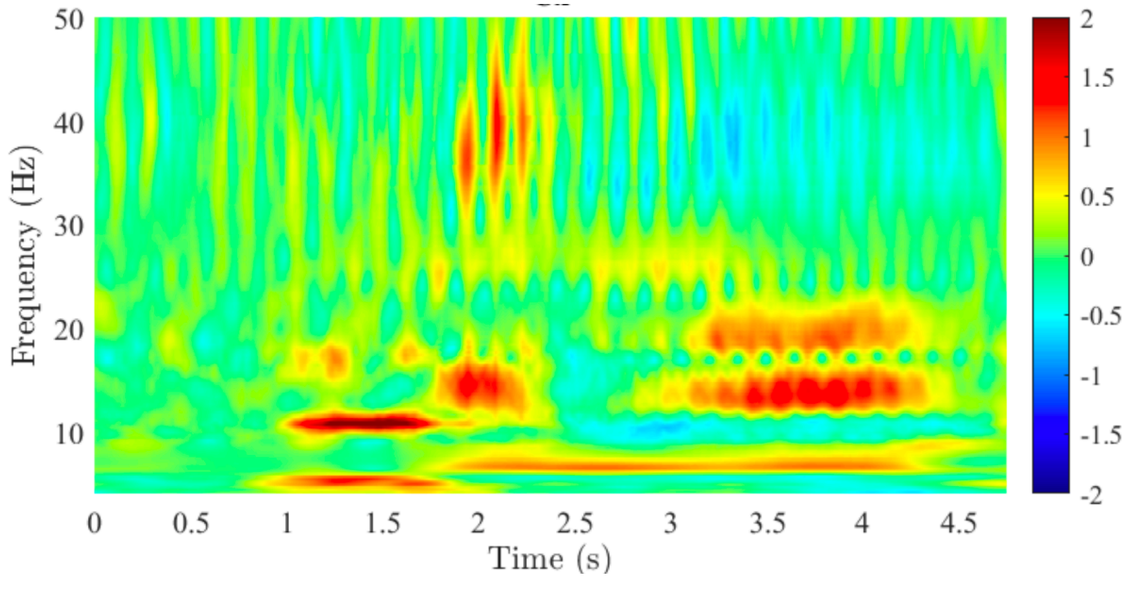


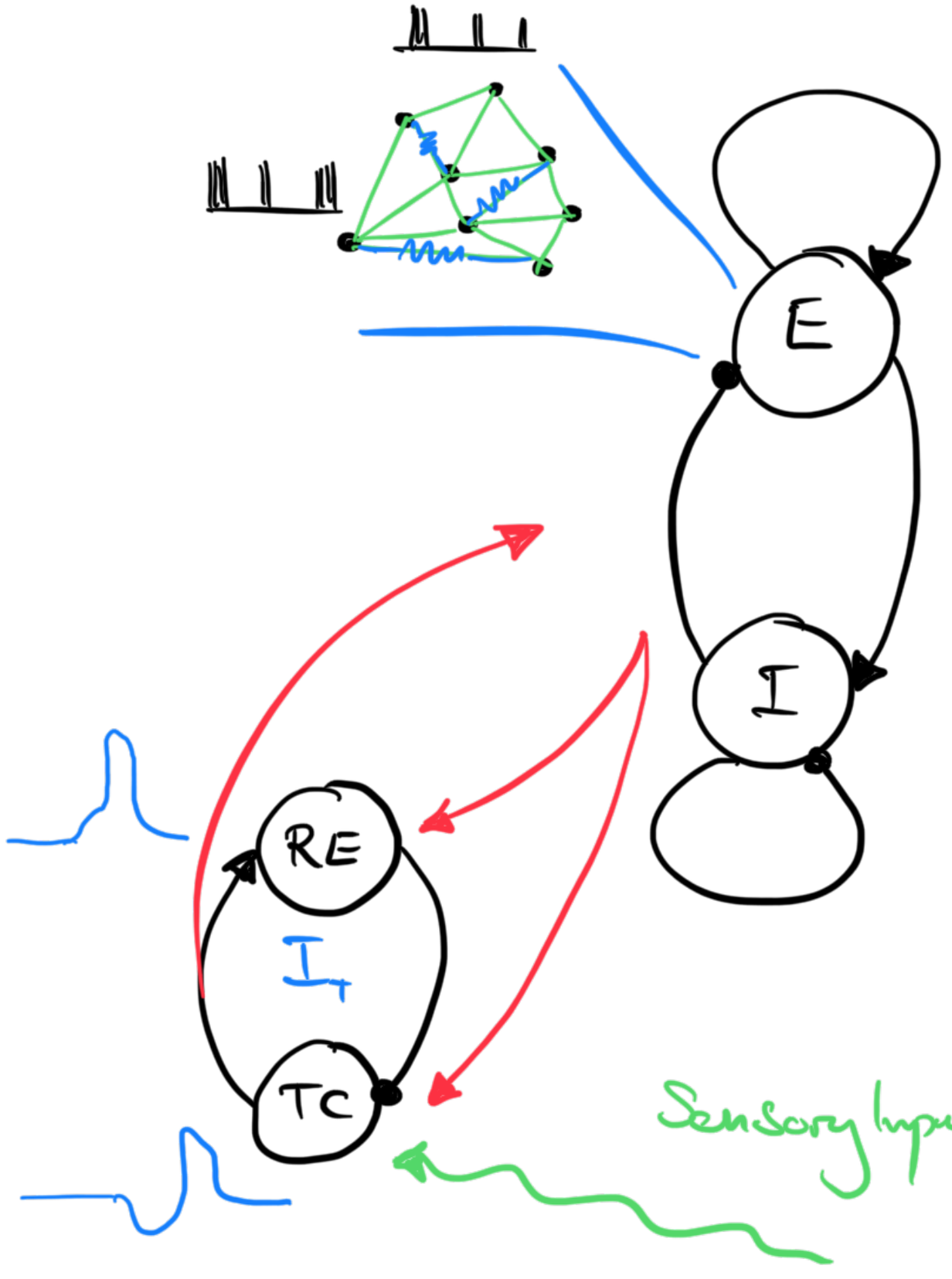
Tourette's



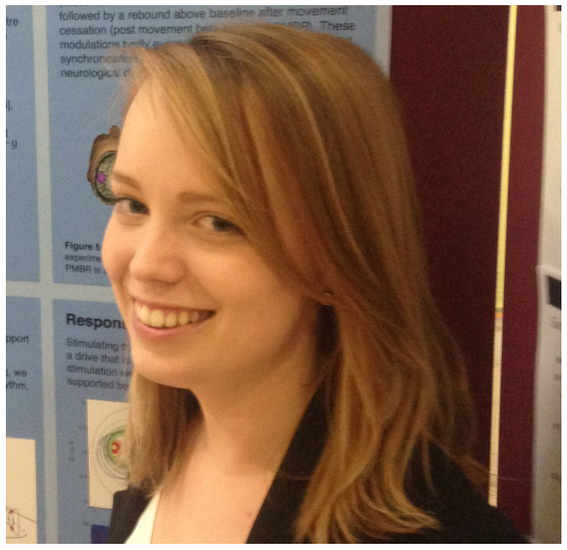
New mean field thalamo-cortical model







Acknowledgements



Áine Byrne (Dublin)

Rachel Nicks

Reuben O’Dea

Michael Forrester

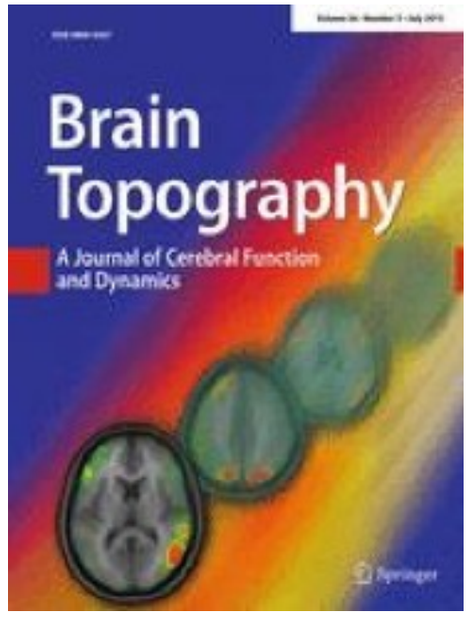
Daniele Avitabile (VU)

James Ross

Robert Allen

Sammy Petros

Mean-field models for EEG/MEG: from oscillations to waves,
Á Byrne, J Ross, R Nicks and S Coombes
Brain Topography, Vol 35, 36-53, 2022



frontiers

Next generation neural population models, S Coombes Frontiers in Applied Mathematics and Statistics, Vol 9, 2023

… and talk to Carlo!

Texts in Applied Mathematics 75

Stephen Coombes · Kyle C. A. Wedgwood

Neurodynamics

An Applied Mathematics Perspective

This book is about the dynamics of neural systems and should be suitable for those with a background in mathematics, physics, or engineering who want to see how their knowledge and skill sets can be applied in a neurobiological context. No prior knowledge of neuroscience is assumed, nor is advanced understanding of all aspects of applied mathematics! Rather, models and methods are introduced in the context of a typical neural phenomenon and a narrative developed that will allow the reader to test their understanding by tackling a set of mathematical problems at the end of each chapter. The emphasis is on mathematical- as opposed to computational-neuroscience, though stresses calculation above theorem and proof. The book presents necessary mathematical material in a digestible and compact form when required for specific topics. The book has nine chapters, progressing from the cell to the tissue, and an extensive set of references. It includes Markov chain models for ions, differential equations for single neuron models, idealised phenomenological models, phase oscillator networks, spiking networks, and integro-differential equations for large scale brain activity, with delays and stochasticity thrown in for good measure. One common methodological element that arises throughout the book is the use of techniques from nonsmooth dynamical systems to form tractable models and make explicit progress in calculating solutions for rhythmic neural behaviour, synchrony, waves, patterns, and their stability. This book was written for those with an interest in applied mathematics seeking to expand their horizons to cover the dynamics of neural systems. It is suitable for a Masters level course or for postgraduate researchers starting in the field of mathematical neuroscience.

ISBN 978-3-031-21915-3

▶ springer.com

TAM 75

Coombes · Wedgwood

Neurodynamics

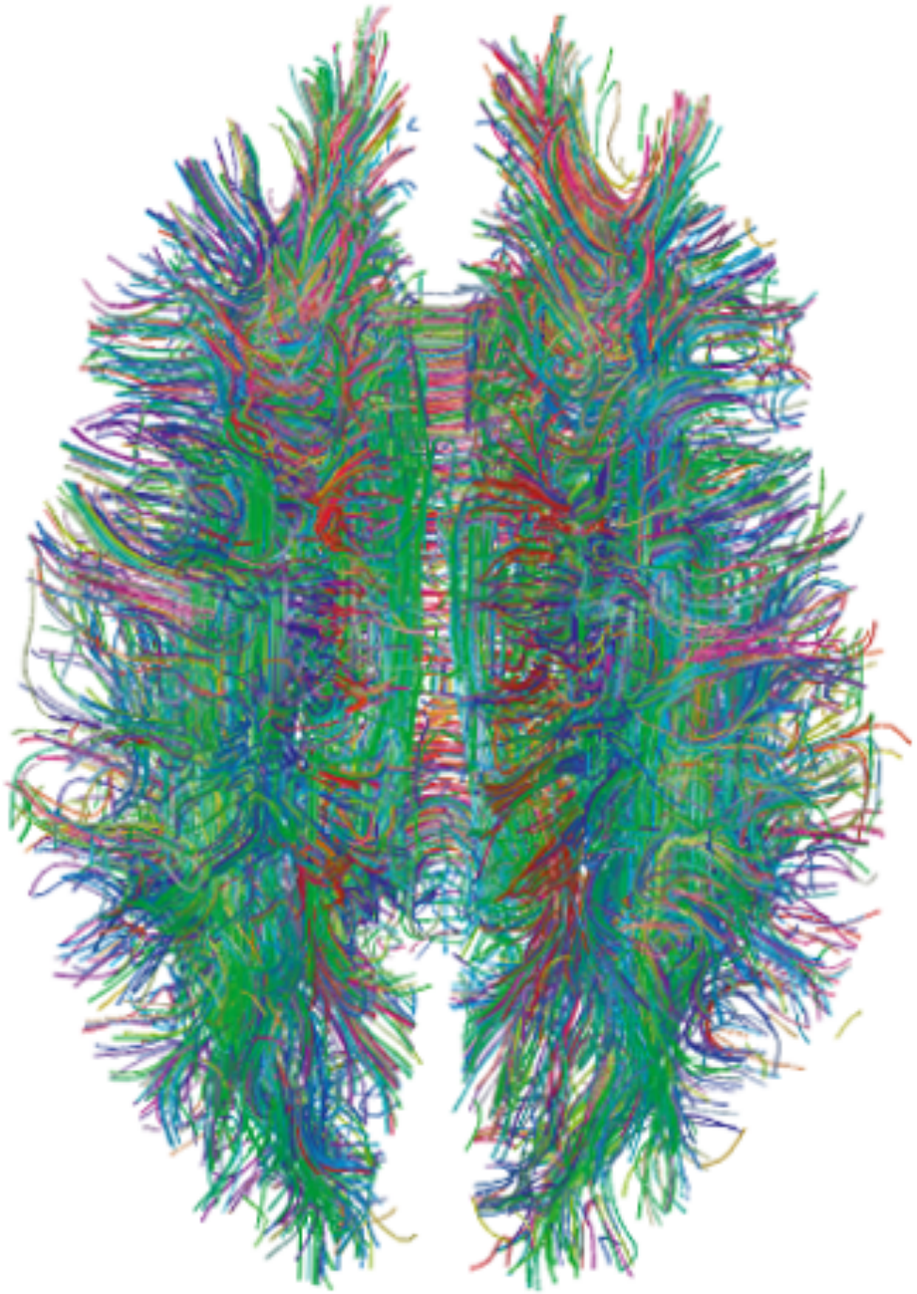
Texts in Applied Mathematics 75

Stephen Coombes
Kyle C. A. Wedgwood

Neurodynamics

An Applied Mathematics Perspective

Springer



Mathematical Neuroscience and Applications

DIAMOND OPEN ACCESS

https://mna.episciences.org/